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Journal
of
The Franklin Institute
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Vol. 188 NOVEMBER, 1919 No. 5

ATMOSPHERIC ELECTRICITY.*

BY

W. F. G. SWANN, A.R.C.S., D.Sc.,

Professor of Physics at the University of Minnesota.[1]

INTRODUCTION.

It had been surmised by several early investigators that lightning and the electric discharge were in some way connected; but, it was not until the middle of the eighteenth century that Franklin, the pioneer whose name this institution bears, showed, by obtaining electrical discharges from a kite, that there was really a very close connection between the two phenomena.

Although the electrical manifestations of the atmosphere attain their most spectacular form in the thunderstorm, nevertheless, during the ordinary quiet days, our air is constantly the seat of electrical phenomena hardly less interesting and puzzling to the student of science.

Our earth is not an electrically neutral body. Its surface is coated with a charge of negative electricity; and the amount of this charge is such as to give rise to a very considerable electrical field in the atmosphere. In the technical mode of expression we say that the potential increases, as we rise above the earth's surface, to the extent of about 150 volts per metre; that is to say, the

* Presented at a joint meeting of the Electrical Section and the Philadelphia Section American Institute of Electrical Engineers, held Thursday, November 1, 1917.

[1] At the time this lecture was given, Chief of the Physical Division, Department of Terrestrial Magnetism, Carnegie Institution of Washington.

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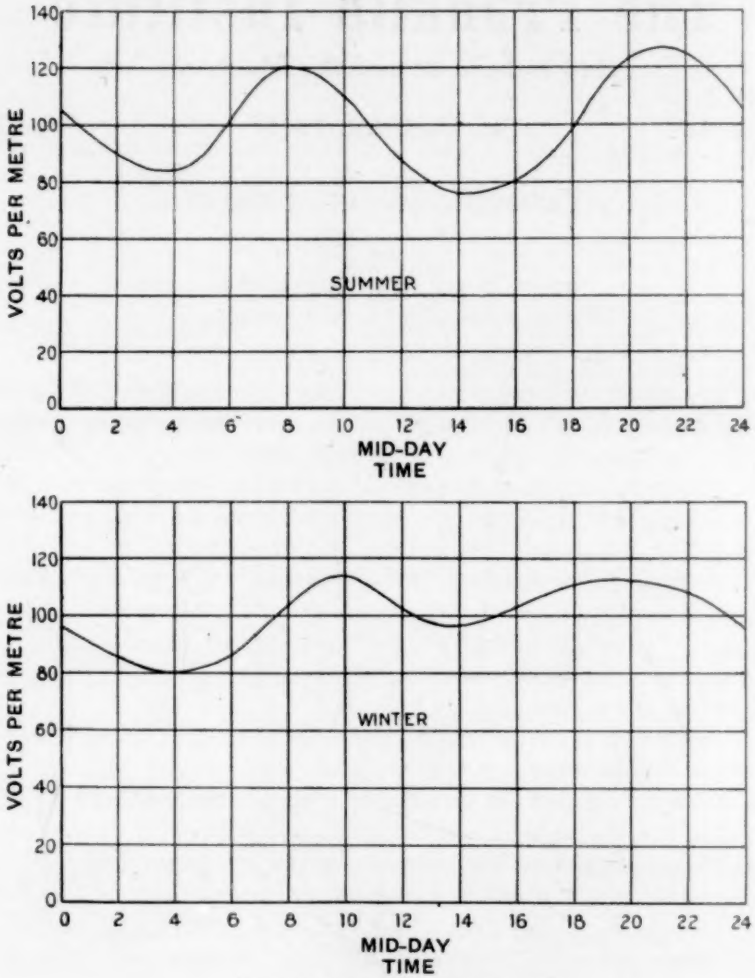
difference in electrical pressure between the earth and a point one metre above its surface is about one and a half times as great as the difference in electrical pressure between the terminals of

FIG. 1.

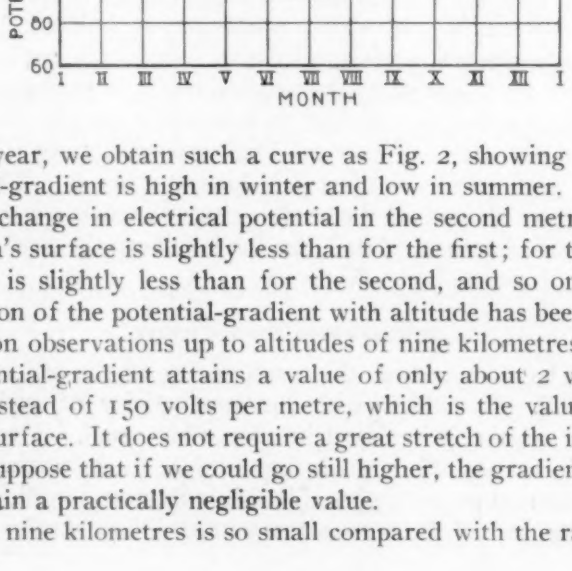


Diurnal variation of potential-gradient

our lighting circuits. The change of electrical potential per metre is called the potential-gradient, and is one of the atmospheric-electric quantities most frequently studied.

Except in abnormal weather, the potential-gradient is almost invariably directed in such a way as would result in negative electricity being repelled away from the surface of the earth; that is, in such a way as to correspond to the existence of a negative charge upon the earth's surface. It is by no means an invariable quantity, however, but goes through a fairly regular series of changes throughout the day, and throughout the year, changes amounting to fifty per cent. or more of its whole value. In Fig. 1 the upper curve is typical of the variation of the potential-gradient throughout the day in summer, and the lower curve is typical for a winter day. If we take the average value of the potential-gradient for each day of the year, and plot this quantity through-

FIG. 2.



out the year, we obtain such a curve as Fig. 2, showing that the potential-gradient is high in winter and low in summer.

The change in electrical potential in the second metre above the earth's surface is slightly less than for the first; for the third metre it is slightly less than for the second, and so on. This diminution of the potential-gradient with altitude has been traced by balloon observations up to altitudes of nine kilometres, where the potential-gradient attains a value of only about 2 volts per metre instead of 150 volts per metre, which is the value at the earth's surface. It does not require a great stretch of the imagination to suppose that if we could go still higher, the gradient would soon attain a practically negligible value.

Now nine kilometres is so small compared with the radius of

the earth that if we were to represent the earth by a circle drawn upon an ordinary page, we should find difficulty in drawing the circle with a line so thin but that its thickness alone would represent more than nine kilometres. We realize how much remains to be known on the purely experimental side of atmospheric-electricity when we recall that practically all of our measurements are confined to a very thin shell of this kind, and indeed, most of them to a very small fraction of this shell. If we could only know something of the conditions over a shell thirty or forty kilometres thick, we should probably be on the high road to the solution of much that is now obscure.

I have said that, within a thin shell of the atmosphere only about ten kilometres thick, the potential-gradient falls from about 150 volts per metre to practically zero. It can be shown as a matter of pure mathematical reasoning from the known laws of electrostatics that the only explanation of this diminution of potential-gradient with altitude is to be found in the supposition that the portion of the atmosphere within the shell holds a positive charge just equal to the negative charge on the surface of the earth; and direct experiment has, of course, confirmed the presence of this positive charge.

Now a state of affairs where a negative charge is to be found on the surface of the earth and a positive charge in the surrounding atmosphere is one which could persist indefinitely if the atmosphere were a perfect insulator. But it is not a perfect insulator. Its power of conducting electricity is extremely small; but, it is nevertheless sufficient to insure that 90 per cent. of the charge on the earth's surface would disappear in about 10 minutes if there were no means of replenishing the supply. The origin of the replenishment of the earth's charge, which we shall presently discuss in greater detail, has for long been the most puzzling problem of atmospheric electricity.

ATMOSPHERIC CONDUCTIVITY.

As I have remarked, the conductivity of the atmosphere is very small. Compared with the conductivity of such a substance as copper, it is almost inconceivably small. A column of air one inch long offers as much resistance to the passage of the electric current as a copper cable thirty thousand million million miles long, and of the same cross-section, *i.e.*, as much resistance

as that of a copper cable long enough to reach from here to Acturus and back twenty times. In view of the very small conducting power of the atmosphere, we might be constrained to imagine that the difficulty is not to explain that conductivity, but rather to explain why the conductivity is not much greater than it actually is. Nevertheless, modern work on the origin of electrical conduction in gases has lead to such definite conceptions of the mechanism of this process that even the conductivity of the atmosphere is not so small as to avoid the necessity of an explanation of its origin.

According to modern views, matter is composed almost, if not entirely, of positive and negative particles or electrons. We know a great deal about the negative electrons. They all carry the same electric charge, and are all of the same size and mass, the radius being about 10^{-13} cm. and the mass about 10^{-27} gramme. The ordinary molecule of oxygen or nitrogen contains as much positive as negative electricity; and, if placed in an electric field, it would show no tendency to move. As a result of certain agencies, however, it is possible to detach a negative electron from a molecule, leaving the remainder positively charged. This remainder constitutes what is called the positive ion. The negative electron may travel freely for some time, but sooner or later it will attach itself to a neutral molecule, or, according to the views of some, to a group of molecules, and so form the negative ion. These ions render the air conducting because, under the influence of an electric field, the negatively and positively charged ions move respectively towards the positively and negatively charged bodies and discharge them.

Now suppose that, in the atmosphere, there is some agency which produces ions continually. The number of ions per c.c. will grow and grow; it would grow indefinitely were it not that the ions which have been produced tend to recombine, positive to negative, by their mutual attraction; and, the more there are present, the more rapidly do they recombine. If the ions are produced at a constant rate, the number per c.c. will grow only until the number which recombine per second is equal to the number produced per second. Thus, to take a concrete example, it appears that, in the atmosphere, about 6 pairs of ions are produced per cubic centimetre each second; and, by the time the number has grown to about 2400 of each kind per c.c., six pairs of ions

will recombine per second, and the total number will cease to grow further.[2]

How, then, are we to account for the constant production of ions in the atmosphere? It appears that, in the lower temperature, two causes are mainly responsible:

(1) The radioactive emanations in the air.

(2) The "penetrating radiation."

Concerning the former, I must first remind you that, according to modern views, a substance like radium is in a continual state of disintegration. The disintegration of radium itself takes place very slowly; about 2000 years must elapse before a given quantity of radium becomes reduced to one-half of its original amount. As the radium decays, it gives birth to a gas called radium emanation. This gas is much shorter lived than radium itself. Half of any given quantity of it disappears in 3.85 days; and as it dies it gives birth to another substance, radium *A*, which again dies, giving rise to radium *B* and so on.

Now the study of the radioactive substances has shown that when one of these atoms disintegrates and gives rise to the next in succession, one or all of three types of radiation are emitted from the atom. First, there is the α particle, a positively charged atom of helium, having a mass equal to four times that of the hydrogen atom, and travelling with a velocity of the order one-tenth that of light, which itself travels with a speed of 186,000 miles per second. Then there is the β particle which is merely an ordinary negative electron, travelling with a velocity comparable with that of light, and having a mass one two-thousandth that of the hydrogen atom. Lastly, there is a type of radiation, the γ radiation, which is generally supposed to be some sort

[2] At this point the lecture was illustrated by an experiment. An insulated conducting cylinder was shielded by an earthed metal cylinder, and connected by means of a wire to a quadrant by a DoleZalek electrometer, whose other quadrant was connected to the electrometer case, which was mounted on an insulating support. From the electrometer mirror a spot of light was reflected to a scale on the screen. By means of a fan a current of air could be drawn through the space between the two cylinders. A battery of 150 volts maintained a difference of potential between the two cylinders, so that if ions were drawn into this region by the fan, those of appropriate sign were driven across to the central cylinder and so caused a deflection of the spot of light.

By means of this apparatus, the ions introduced into the air entering the cylinders, by means of a match flame, and a tube of radium held over the open end were made evident.

of an ether pulse, but not primarily of as material a nature as the α or β particle.

Now all of these radiations possess the power of ionizing, either directly or indirectly, a gas through which they pass. The α particles are very efficient ionizers; and, so busily do they set about their work, that they expend all their available energy in this process and come to rest in passing through only a few centimetres of air. The β particles are much less efficient ionizers. They pass through the gas with but little commotion, and may travel several metres before coming to rest. The γ rays are still more penetrating and less efficient ionizers than the β particles; and their intensity still retains 36 per cent. of its initial value after they have passed through 100 metres of air.

Now the soil contains radium and other substances of this nature, and these give rise to active gaseous emanations which diffuse out into the atmosphere. During the disintegration of these emanations in the atmosphere, α, β, and γ rays are emitted with the result that the air becomes ionized and is rendered conducting. The amount of radium emanation in the atmosphere varies very much from time to time. It is, however, always extremely small. In a shell of the atmosphere extending over the whole of the earth's surface and comprised between that surface and an altitude of one kilometre, we shall find only about 250 grammes of radium emanation. Or, expressing this magnitude in another form, we may say that each cubic centimetre of the atmosphere contains on the average only between one and two molecules of radium emanation, as compared with the thirty million million million molecules of air which it holds. Nevertheless, by adding up the ionization produced by the α, β, and γ rays from the emanation and its products we are able to account for the production of 1.7 ions per second in each cubic centimetre of the atmosphere. In addition to radium emanation, another radioactive gas, thorium emanation, is also to be found in the atmosphere and contributes to the ionization. Again, a certain amount of atmospheric ionization is attributable to the radioactive materials in the soil. Here we are only concerned with the γ ray ionization, since the α and β rays are so readily absorbed that they never succeed in getting out of the soil. The soil contains, on an average, about 4×10^{-12} gramme of radium per c.c., and, by calculating the amount of γ radiation which can be

accounted for, after allowing for the absorption of the rays coming from different depths in the soil, we find enough to provide for the production of 0.80 ion per c.c. per second. In a similar way, the thorium in the soil is found to be capable of accounting for a rate of production of 0.80 ion per c.c. per second.

TABLE I.

Ionization of the Atmosphere by Radioactive Material.

Type	Whence	Radium	Thorium	Total
α rays	Air	1.63	1.00	2.63
β rays	Air	0.035	0.025	0.06
γ rays	Air	0.035	0.025	0·06
γ rays	Earth	0.80	0.80	1.60
		2.50	1.85	4.35

These results on the amount of ionization which the radioactive material is capable of accounting for are summarized in the accompanying table taken from a paper by A. S. Eve [3]; and it appears that, altogether, a rate of production of 4.35 ions per c.c. per second can be accounted for in this way. From a knowledge of the rate at which ions recombine we can calculate the number to which they would build up in the atmosphere on account of a rate of production of 4.35 ions per c.c. per second, and the result comes out to 1320 ions of each sign per c.c.

Measurements of the numbers of ions per c.c. in the atmosphere are attended with considerable difficulty, for all sorts of different types are present. In particular, there is a class of ion which is very sluggish in its motion, and which is probably nothing more than an ordinary ion which has attached itself to a dust particle. On account of their sluggishness, these ions contribute very little to the conductivity, but they nevertheless influence the requirements in the matter of rate of production of ions; for, they contribute to the rate at which the ions recombine. It has been customary to measure the numbers of ions per c.c. by a method which takes account of the most mobile ions only, and it appears that about 800 pairs of these exist per c.c. Thus, quite apart from the direct evidence of the existence of the sluggish ions, we may conclude that ions of this type must be present,

[3] "On the Ionization of the Atmosphere due to Radioactive Matter." *Phil. Mag.*, S. 6, vol. xxi, pp. 26–40, 1911.

since the radioacitve material in the air and soil is alone capable of accounting for 1320 ions per c.c.

The uncertainty of our knowledge of the true average values for the total numbers of ions of all kinds, and of the appropriate rates of recombination, prevents us from doing more than to say that, as far as measurements on land are concerned, the radioactive material in the air and earth is sufficient to account for the whole of the ionization produced. A difficulty presents itself, however, when we consider the results of observations made over the sea. Those who have made observations over the ocean have found very little radioactive material. Some of the most recent observations have been made on the yacht *Carnegie*, owned by the Carnegie Institute of Washington; and, as a result of observations extending over about 50,000 miles in the Pacific and Sub-Antarctic Oceans, an average radioactive content was found, for the atmosphere, amounting to only 2.5 per cent. of that found on land, and the radioactive content of the sea water collected in regions remote from land was immeasurably small. Nevertheless, the number of the more mobile ions found per c.c. over the ocean is as great or greater than that found over land, as will be seen from Table II, which represents a comparison of the results of the *Carnegie's* fourth cruise with those obtained by other observers at sea, and with land values. The number of ions is much larger than can be accounted for by the small quantity of radioactive material in the ocean air.

TABLE II.

Comparison of Land and Sea Values of the Ionic Content.

Nature of observations	Number of positive ions per c.c.	Number of negative ions per c.c.
Mean of land observations obtained by various observers	737	668
Mean of ocean values for the fourth cruise of the *Carnegie*	804	677
Mean of former ocean values obtained by various observers	736	588

What, then, is the agency responsible for the ionization over the ocean? It appears to be the so-called "penetrating radiation." If a hermetically sealed vessel is freed from radioactive air, we nevertheless find that ions are produced in it at a fairly

constant rate of about 10 ions per c.c. per second over the land. An appreciable proportion of this ionization is due to the γ ray radiation which comes from the soil and passes through the walls of the vessel. That the whole of it is not due to this cause is, however, borne out by several circumstances. In the first place, ionization in closed vessels is found to take place over the ocean, where there is practically no radioactive material, and it there amounts to about 4 ions per c.c. per second in a copper or a zinc vessel. Secondly, if, in experiments on land, the apparatus is surrounded by a wall of water of sufficient thickness to shield off practically completely the γ ray radiation from the earth, there still remains a rate of production of about 4 ions per c.c. per second. But the strongest evidence of the reality of the penetrating radiation is to be found in the results obtained in balloon ascents by W. Kolhörster.[4] It appears that, with increase of altitude, the ionization within a closed vessel at first diminishes up to an altitude of 700 metres. This we should expect as a result of the absorption of the earth's γ ray radiation by the atmosphere. Above this altitude an astonishing thing happens, however. The ionization commences to increase, and goes on increasing with greater and greater rapidity until, at an altitude of 9000 metres, the intensity of ionization is in excess of that at the earth's surface by about 80 ions per c.c. per second. An increase of 20 ions per c.c. per second takes place in the last kilometre, and the rapidity of the increase at these higher altitudes is such as to suggest that very large values of the ionization would be obtained at altitudes not very much greater.

It thus appears that there is some source of ionization other than the radioactive materials in the soil and lower atmosphere; and this agency, whatever its origin, appears to be the sole source of ionization over the ocean. The rate of production of ions must certainly be greater over the land than over the ocean by the amount attributable to the radioactive materials on land. It is probable, however, that over the land, where dust nuclei are more plentiful than over the ocean, a much larger proportion of the ions produced join the slowly moving class than is the case with the ions produced over the ocean; and it is to this cause that we must attribute the fact that the ionic density for the more mobile ions is no greater over the land than over the sea.

[4] Aerophysikalischer Forschungsfonds Halle, Abhandlung 14.

To return to the remarkable increase with altitude shown by the ionization within closed vessels. Such a variation at once suggests a radiation coming from some source external to our globe, or from some active agency in the upper regions of the atmosphere, the decrease in intensity encountered as we *descend* into the atmosphere being accounted for by absorption. The rapidity of the absorption can be calculated from the observations on the variation of intensity with altitude; and, it appears that the observations require us to assume for the radiation a penetrating power ten times that of the most penetrating γ rays known in radioactive substances. We must not be too greatly skeptical as to the possibility of the existence of so penetrating a radiation, for light itself is extremely penetrating as regards air, since we can see the stars through the whole thickness of our atmosphere. The "penetrating radiation" is not light, however, for it can pass through the walls of a metal vessel. Its true origin remains one of the most interesting speculations of atmospheric electricity. Linke has suggested a layer of strongly radioactive cosmical dust in the atmosphere at an altitude of 20 kilometres. For my own part, it seems more natural to seek an explanation from another stand-point.[5]

It is generally supposed that the Aurora is due to light generated by the impact, with our atmosphere, of negative electrons shot from the sun. The stream of electrons is not confined to the sunlit side of the earth, since the paths of the electrons are bent in passing through the earth's magnetic field, and some enter our atmosphere on the side remote from the sun. Now it is a well-established fact that when electrons strike molecules of matter X rays are produced. The greater the velocity of the electrons, the higher the penetrating power of the X rays produced. γ rays themselves are nothing more than a particularly penetrating type of X rays. Modern developments in our knowledge of X rays and γ rays have taught us how to calculate the velocity which an electron must have in order to produce γ rays of any given degree of penetration. Now, from considerations of the theory of the Aurora, into which time will not permit me to enter, Birkeland has shown that it is necessary to assume that the electrons which are responsible for this phenomenon have an energy enormously

[5] The paragraph following this remark was not included in the lecture as given.

greater than that of even the swiftest β rays with which we are familiar in the laboratory; and, if we invoke the assistance of these high-speed electrons necessitated by the theory of the Aurora, we find that their speed is sufficiently great to account for the production of a γ ray radiation of a degree of penetration fully as great as that of the "penetrating radiation." The electrons themselves are not the "penetrating radiation" for, in spite of their great energy, they could not travel right through our atmosphere. By conversion of their energy into the γ ray type, however, a radiation of much greater penetrating power is created.

ATMOSPHERIC-ELECTRIC MEASUREMENTS.

Time will not permit an extensive discussion of atmospheric-electric measurements; but a general survey of this field may perhaps best be given by a very brief description of the procedure adopted on the *Carnegie's* fourth cruise, in respect of which it fell to my lot to plan the work and devise the instruments and methods of measurement.

The quantities measured were: The potential-gradient, the conductivities arising from the positive and negative ions, the numbers of positive and negative ions per cubic centimetre, the radioactive content of the atmosphere and of the sea water, the numbers of pairs of ions produced per c.c. per second in a hermetically sealed vessel, and the diurnal variations of the potential-gradient and ionic density. The ionic mobilities were calculated from the results for conductivity and ionic density, and, in addition, the pressure, temperature, and humidity were measured.

Electroscopes.—All the measurements require, at some stage of the operation, the use of an electroscope. It will be recalled that the ordinary type of electroscope consists of two gold leaves suspended on a rod which is fixed in an insulating plug, and surrounded by a metal case. This type is quite unsuitable for use at sea, on account of the disturbing influence of the motion of the ship. Within recent years, however, another type of electroscope has been devised, in which the controlling force is brought about by a method independent of gravity. In one form of the instrument, the bi-filar electroscope of Wulf, the gold leaves of the older electroscopes are replaced by two quartz fibres coated with platinum to render them conducting. At their upper ends, the fibres are soldered to the main terminal of the instrument, and

their lower ends are attached to a quartz bow whose ends are fixed to a frame. When the fibres are charged, they repel each other, and the resulting motion, which can be read by a microscope with a scale in the eye-piece, is resisted by the quartz bow. This instrument is very convenient where sensitivities of the order of magnitude of 2 volts per division, and working ranges of about 20 to 250 volts are required.

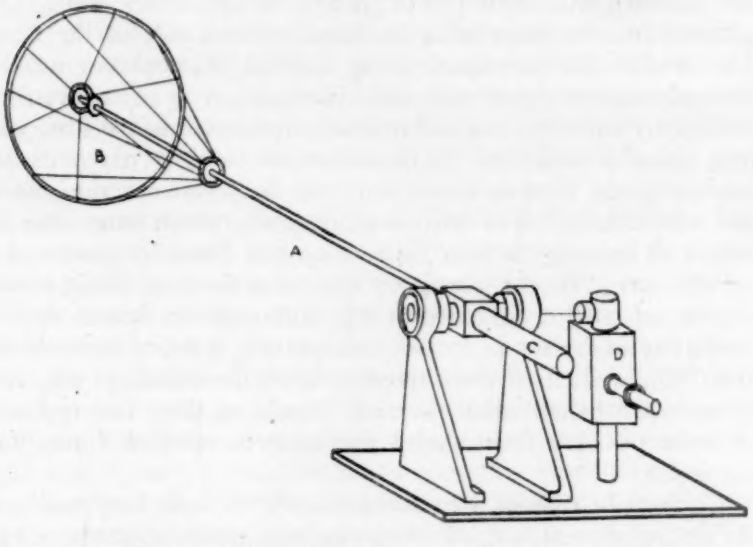
Another convenient form of quartz-fibre electroscope of a design devised by Wulf, comprises a single platinized quartz fibre attached at its lower end to a quartz bow and at its upper end to the main terminal of the instrument. Two insulated metal plates are mounted with their planes parallel to each other and to the quartz fibre, one plate being mounted on each side of the fibre. The case of the instrument being earthed, these plates may be charged respectively to +100 and −100 volts, or to any convenient amount by batteries, and a charge communicated to the fibre will then cause a deflection. The deflection for a given potential applied to the fibre increases with the field between the plates, and with diminution of tension on the fibre, which latter may be varied by moving the bow support up and down by means of a suitable screw. In the laboratory it is not difficult to obtain a sensitivity of 100 divisions per volt, although on board ship a sensitivity of from 5 to 10 divisions per volt is found more desirable. On account of their freedom from the effects of tilt, and on account of their small electrical capacities, these two types of electroscope have been found the most convenient forms for use at sea.

Potential-Gradient.—In former work on both land and sea, the customary method of measuring the potential-gradient has been the following:

An insulated rod is caused to project from the wall of a house, or the rail of the ship, as the case may be, and, to the external end of the rod, a plate coated with radioactive material is attached. The other end of the rod is attached to an electrometer or electroscope. Now, on account of the potential-gradient, there is ordinarily a difference of potential between the wall of the house (or the rail of the ship) and a point some distance away from it. The radioactive plate renders the air in its immediate vicinity conducting, and electricity consequently flows to the rod until it attains the potential of the air at its external end, as determined

by the electrical field of the earth, modified by the presence of the apparatus itself. The quantity measured by the electrometer may be looked upon as proportional to the potential-gradient; and, in order to obtain absolute values, it is necessary to make a set of measurements simultaneously over some flat surface in such a manner that the measurements are independent of the distortion brought about by the apparatus itself. The chief disadvantages associated with this form of apparatus for work aboard ship lie in the slowness of its action, which necessitates very perfect insulation if accurate results are to be obtained. Further, since

FIG. 3.



the motion of the ship continually alters the distance between the end of the rod and the water, the potential recorded can only be looked upon as a sort of average. In order to overcome these difficulties, the apparatus illustrated in Fig. 3 was constructed. The instrument consists of a brass tube *A* fixed at one end to an axle, so that it can rotate in a plane containing the fore-and-aft line of the ship. The axle is mounted on supports fixed to the stern rail of the ship, and the projecting end of the brass tube carries a disc *B*, made in the form of a skeleton of a parasol, and covered with gauze not shown in the figure. The handle *C*, by which the rotation is brought about, is insulated from the axle.

and the latter is itself insulated from earth by causing it to work in brass tubes fixed into their supports with sulphur insulation. The axle is connected by a thin wire to a Wulf bi-filar electroscope *D*, the wire and axle being in the same line. It is arranged that, when the brass tube is vertical and the parasol attachment downward, the electroscope system is earthed. On rotating the tube to some other position, fixed by a stop, a deflection proportional to the potential-gradient is obtained in the electroscope. Insulation difficulties are entirely overcome, since the leakage occurring during the turning of the handle from one position to another is negligible. Further, the operation can be performed so quickly that readings may always be taken at the same position of tilt of the ship as determined by a spirit level.

Conductivity and Ionic-Content.—The conductivity contributed to the atmosphere by the ions of either sign is the product of the number of these ions per c.c. the charge on an ion, and the ionic mobility, which is the velocity which the ion would attain under unit electric field. The method which is usually employed for measuring the conductivity of the air is that due to Gerdien. In this method, air is drawn by a fan through the space between two concentric cylinders, the central member of which is charged and connected to an electroscope. The theory of the instrument shows that, so long as the velocity of the air-current is large enough to insure that the central cylinder is unable to extract from the air all of the ions which it attracts as the air passes through, the rate of loss of charge by the cylinder is independent of the air velocity, and depends only upon the conductivity contributed by the ions of sign opposite to the charge on the central cylinder. Under these conditions, measurements of the potential of the central cylinder at two different times during the passage of the air current, combined with the knowledge of certain electrical capacities associated with the instrument, enable the conductivity in question to be deduced. If the velocity is so small that all of the ions of sign opposite to that on the central cylinder are drawn thereto as the air passes through, the indications of the instrument do depend upon the velocity of the air; in fact, the rate of loss of charge is proportional to the velocity of the air under these circumstances, since the air is robbed of all of its ions as it passes through the space between the cylinders. A knowledge of the rate of flow of the air then enables one to deduce, from the

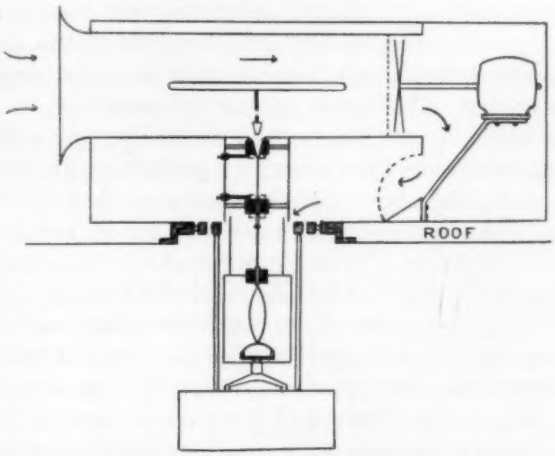
indications of the instrument, the number of ions per cubic centimetre. The chief disadvantage of this type of instrument for measuring the ionic densities lies in its lack of sensitivity. In the usual form of instrument, the central cylinder is connected to a Wulf electroscope of the bi-filar type reading up to about 200 volts, and the outer cylinder is earthed. The rate of movement of the fibres when the air flows through the instrument under these conditions is extremely slow, and as much as half an hour may be necessary to obtain a single satisfactory observation. The result of this is that, in some measurements—for example those of ionic mobilities, in which it is necessary to assume constancy of the ionic content of the atmosphere over the period of two measurements—the departure from this condition is frequently sufficient to cause the quantity measured, which is essentially a positive quantity, to come out negative in the calculations.

In order to increase the sensitivity and rapidity of action of the instrument, I devised a modified form, in which the central cylinder of the apparatus is connected to the fibre of a unifilar electroscope of the Wulf type, the sensitivity of which is made as great as is desired. The potential of the fibre is never allowed to depart far from zero, and the necessary field is obtained by insulating and charging the outer cylinder to about 150 volts. On releasing the fibre from earth it, of course, starts to move at a rate determined by the rate of supply of electricity to the central system from the air. The fact that the central system is never allowed to depart far from zero potential also enormously reduces the leakage error, which otherwise would become a very serious consideration, especially in work at sea. Indeed, leakage may be entirely compensated by starting observations with the fibre charged in such a sense that it crosses the zero point during the observation, for it may then be arranged that the fibre readings which are chosen as the bases of the measurements lie at equal distances on each side of the zero. In order to avoid an alteration of the number of ions entering the apparatus, by the charge on the outer cylinder, the latter is surrounded by yet another cylinder which is insulated from it and earthed. Even this does not entirely overcome the difficulty, however, as will readily be surmised when it is remembered that the potential of a point a short distance outside the cylinder is zero, and that of a point a short distance inside is about 150 volts, so that the ions, in order to get

in, have to move in opposition to electric forces. In order to completely eliminate this difficulty, the central cylinder is provided with an additional attachment which it is unnecessary to describe here.[6]

Except in matters of detail, the method of measuring the conductivity on the *Carnegie* is the same as that due to Gerdien. The fan is, however, driven by an electric motor instead of by hand, so that a larger current of air is obtained, and greater sensitivity can be secured by the employment of a higher potential on the central system than would be otherwise possible. The apparatus is illustrated in Fig. 4.

FIG. 4.



Since good insulation is essential to such measurements as these, it is desirable that the parts of the apparatus which contain insulating materials shall be well protected. For this reason, a small observatory has been erected aboard the *Carnegie,* and this observatory houses the apparatus for the measurement of conductivity, ionic density, penetrating radiation, and part of the apparatus for the determination of the radioactive content of the atmosphere. It is arranged that the cylinders which receive the air project through the roof of the observatory, but the electroscope systems are mounted permanently inside. The advantage of this arrangement lies in the fact that, in so far as the temperature of the inside of the observatory is always above the dew

[6] *Terr. Mag.,* vol. xix, p. 171, 1914.

point of the air outside, condensation on the insulated parts is less likely to occur than if the apparatus were exposed to the open air.

Measurement of the Penetrating Radiation.—For the measurement of the number of pairs of ions produced per cubic centimetre per second in a closed vessel, an air-tight copper vessel of 27 litres capacity is employed. A rod which is insulated from the copper vessel is connected to the fibre of a unifilar electroscope. The copper vessel is charged to a potential sufficiently high to cause saturation for the ionization produced inside, so that on releasing the fibre from earth, the central system charges up at a rate determined by the rate of production of ions in the vessel. The insulating material separating the central rod from the vessel is, of course, divided into two parts by a guard ring which is earthed, thus minimizing leakage from the copper vessel across the amber. The whole apparatus is mounted on a gimbal.

Measurement of the Radioactive Content of the Atmosphere.—In measurements of the radioactive content of the atmosphere, former investigators have largely used a method due to Elster and Geitel. This method, which depends for its action upon the fact that the products of disintegration of the radium emanation in the air are for the most part positively charged, consists in allowing a long negatively charged wire to collect active material from the atmosphere for a period of two hours, and then measuring the saturation current which this active material produces when wound on a frame and introduced into an ionization chamber. In this way are obtained quantities from which a sort of comparison may be made of the amounts of radioactive material in the atmosphere at different places. The theory of the method shows, however, that, apart from the fact that the results given are only relative, considerable uncertainty attaches to their interpretation.

Undoubtedly one of the best methods of determining the radium-emanation content of the atmosphere is that which utilizes the fact that when air containing emanation is drawn over cocoanut charcoal, the emanation is absorbed. The emanation is subsequently expelled by heating, and is measured by its ionizing action. The time required in carrying out this method, and the nature of the apparatus necessary, is such as to render the method impracticable for use aboard ship. The method employed on the

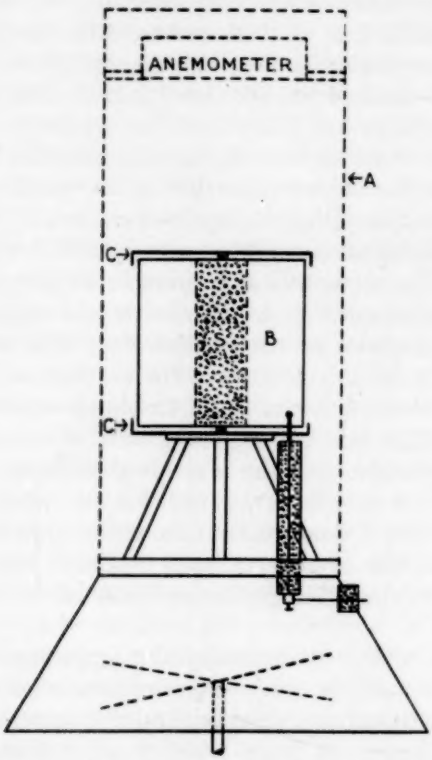
Carnegie consists in drawing air between two concentric cylinders the central one of which is charged negatively to such a high potential that all of the active carriers entering the concentric cylinders are brought to the central system. The saturation current produced in an ionization chamber by the active deposit collected in a given time, combined with a knowledge of the air-flow during the deposit, enables the amount of active material per cubic metre of air to be estimated. The considerations and calculations involved in the deduction of the absolute value of the radioactive content are too complex to be given here in detail. The general principle of this method has been used in one form or another by several investigators on land, and the principal modifications which have been introduced in the apparatus on the *Carnegie* have been made with the object of rendering the results more susceptible of accurate theoretical interpretation, and of increasing the sensitivity of the apparatus and its adaptability for use at sea.

The collecting apparatus as at present employed is shown in its essential features in Fig. 5. It consists of a copper cylinder *A*, 64 cm. long and about 20 cm. in diameter, with an anemometer at one end and a fan at the other. The central system consists of an insulated wooden cylinder *B*, 12 cm. long, supported by a rod passing through its axis and insulated from it by sulphur *S*. The surface of the wooden cylinder is covered by copper foil, held on by rubber bands, and it is on this foil that the deposit is collected. Earthed metal caps *CC* attached to the central rod fit over the top and bottom of the central cylinder without touching it, and insure that the deposit of the active material is confined to the copper foil.

A large air current is necessary if a large amount of deposit is to be obtained; and, in order to secure saturation with a reasonably low potential on the central cylinder it is necessary that the latter shall be large. A large cylinder when subsequenly introduced into the ionization chamber, so as to form the central system there, would reduce the sensitivity in the ionization measurements, however, on account of the large capacity. For this reason, the central system of the ionization chamber is formed of a thin rod, and the foil, after removal from the inner cylinder of the collecting apparatus, is bent over and made to line the walls of the ionization chamber, with the active surface facing inwards. In this way the foil does not contribute to the capacity of the system. The height of the ionization chamber is about twice that

of the foil cylinder, so that the latter covers only the central portion of the wall of the chamber. In this way are avoided difficulties arising from uncertainties which would otherwise be introduced in a knowledge of the fraction of the α particles travelling any assigned portion of their range in the vessel. The central system of the ionization chamber is attached to a unifilar electro-

FIG. 5.



scope, and the potential is applied to the outer vessel, the whole being mounted on a gimbal.

The radioactive content of the sea water is obtained by evaporating samples of the water to dryness and sending the residues to Washington, where the activities are determined.

It has been my object to give a brief description of the methods and appliances used on the *Carnegie's* fourth cruise as an illustration of the general processes which one must employ in work of

this kind. I will not, however, here enter into a general discussion of the results of that cruise, for, to do so would devote, to a special field, too great a proportion of a lecture in which the subject of atmospheric electricity in general is the main theme. I have given a discussion of the results, and a more complete account of the instruments and methods in a paper "Results of Atmospheric-Electric Observations Made Aboard the *Galilee* (1907–1908), and the *Carnegie* (1909–1916)," [7] to which I will refer those interested in fuller information on the subject. I cannot leave this matter, however, without paying a tribute to the energy and enthusiasm of the various observers, who, in spite of the most severe climatic conditions, for example in the tempestuous regions of the sub-antarctic oceans, labored with such faithfulness to make the atmospheric-electric work of the cruise a success.

THE MAINTENANCE OF THE EARTH'S CHARGE.

I now come to the all-important question of the origin and maintenance of the earth's charge. The earth is continually losing negative electricity owing to the action of the potential-gradient operating in the conducting atmosphere. The current from a square centimetre of the earth's surface is only about 2×10^{-16} amperes. The current from the whole earth is only about 1000 amperes, or about as much as is taken by 3000 incandescent lamps. It is nevertheless sufficient to insure that 90 per cent. of the earth's charge would disappear in ten minutes if there were no means of replenishing the loss. How then is the loss replenished?

One of the earliest suggestions was made by G. C. Simpson, who supposed, tentatively, that the sun emitted positive and negative corpuscles of a very high penetrating power. He further supposed that the penetrating power of the negative corpuscles was greater than that of the positive corpuscles, and sufficient to enable them to reach the earth's surface, while the positive corpuscles were caught by the atmosphere. In this way the attempt was made to account for the negative charge of the earth and the positive charge of the atmosphere.

[7] From Publication No. 175 (vol. iii) of the Carnegie Institution of Washington. This paper, which is published in the joint names of the Director of the Department of Terrestrial Magnetism and the present writer, contains, in addition, a compilation of the reports of observers on previous cruises of the *Carnegie* and *Galilee*.

As a matter of fact, it is unnecessary to go to any special pains to account for the positive charge in the atmosphere provided that we can account for the maintenance of the negative charge on the surface of the earth. For, it is an experimental fact that the atmospheric conductivity increases with altitude until its value at an altitude of nine kilometres is about thirty times the value at the earth's surface. It follows from this that if the potential-gradient were the same at an altitude of nine kilometres as it is at the earth's surface, more negative electricity would be driven outwards through a sphere at the altitude of nine kilometres and concentric with the earth than would be driven out of the earth's surface into the atmosphere. The shell of air below the altitude in question would consequently acquire a positive charge, so that the potential-gradient at the altitude of nine kilometres would become less than that at the earth's surface. The process would continue until the diminution of potential-gradient with altitude was just sufficient to compensate for the increase of conductivity with altitude so as to leave the conduction current-density independent of altitude.

The chief objection to Simpson's tentative theory, however, is one which he himself supports in a recent publication,[8] viz., that it postulates a degree of penetration for the corpuscles so enormously great compared with any corpuscular penetration we have become acquainted with in the laboratory, since the suggestion was put forward, that one hesitates to make this hypothesis without further evidence of the existence of such a degree of penetration.

An early suggestion as to the origin of the earth's charge came from C. T. R. Wilson. It is known that, under suitable conditions, ions may act as nuclei for the promotion of condensation of water vapor; and, since water condenses more readily upon the negative than upon the positive ions, Wilson argued that rain should be, on the average, negatively charged. It is true that rain is charged; but it is charged sometimes negatively and sometimes positively; and, unfortunately, it appears that 75 per cent. of the charge brought down by rain is of the positive sign; that is, of a sign opposite to that required by the theory. Apart from this, however, there are several fatal objections to a theory of this

[8] "Some Problems of Atmospheric Electricity." *Monthly Weather Review*, vol. xliv, pp. 115–122, 1916.

kind. In the first place, the fundamental hypothesis as to the condensation of water vapor upon the ions cannot be substantiated in the case of the atmosphere; for theory requires that, in order that water vapor shall condense upon an ion, and grow to a drop of appreciable size, there shall be at least a four-fold degree of saturation, and such a condition of affairs is never attained in the atmosphere. Again, if we suppose the replenishment to take place through the agency of rain, a positive charge equal to the negative charge which has been given to the earth must remain in the particular portion of the atmosphere from which the rain happens to have come at the time. This positive charge will hold the corresponding negative charge at the portion of the earth's surface immediately below it, so that elsewhere, at places where rain is not falling, there will be no potential-gradient. The difficulty in respect of this latter matter is slightly reduced if we postulate the existence of a very highly conducting layer in the upper atmosphere. For, in this case, a separation of opposite charges between earth and cloud will tend to bring about, by induction, a difference of potential between the highly conducting layer and the earth's surface; and this resulting difference of potential will become distributed equally over the whole surface, on account of the high conductivity of the layer.

A theory which has attracted a great deal of attention is one originated by Elster and Geitel, and subsequently modified by Ebert. If ionized air is allowed to pass through a fine tube, it is found, under certain conditions, that the negative ions diffuse to the walls of the tube more rapidly than the positive ions, with the result that the air which emerges is positively charged. Applying this idea to the atmospheric-electric problem, Ebert argued that the air within the pores of the ground must be highly ionized on account of the radioactive material in the soil, and that during the periods of falling barometric pressure this air would come out of the pores into the atmosphere, acquiring a positive charge in the process on account of the smaller coefficient of diffusion of the positive ion. In this way Ebert supposed that the earth would acquire a negative charge and the atmosphere a positive charge. It is, at first sight, a little difficult to see how the positive charge could rise to any appreciable altitude in opposition to the attraction of the negative; but, in order to account for this, Ebert invoked the assistance of rising currents of air. The theory has

been severely criticized by G. C. Simpson, principally on the basis of conclusions drawn from his laboratory experiments, to the effect that the charging influence arising from the diffusion phenomenon would be far too small to account for the required effect.

Again, according to the theory, there should be an upward convection current of positive electricity equal in total amount to the negative conduction current. By utilizing data on the density of the positive charge in the atmosphere, and on the upward convectional velocity, Gerdien has concluded that the latter is far too small to account for the necessary convection current. Quite apart from these considerations, however, it may be shown that,[9] even if the process did take place according to the method supposed by Ebert, the rising charge of positive electricity would become absorbed by the action of the negative conduction current by the time the air had risen to a comparatively small altitude, so that practically the whole of the positive charge in the atmosphere would be confined to a comparatively thin shell (of thickness less than a kilometre for example) near the surface of the earth. The potential-gradient would consequently attain a sensibly zero value at an altitude less than one kilometre, and this we know to be contrary to the facts.

Other objections to the theory are that it provides no satisfactory explanation for the existence of the potential-gradient over the sea, where there is certainly no diffusion phenomenon to support the process. Neither does it provide satisfactorily for the continuation of the process during the periods when the barometer is not falling. In common with all theories which attempt to account for the phenomena by supposing a supply of negative to the earth and positive to the atmosphere at one place, it is confronted with the difficulty that the positive in the atmosphere tends to hold the negative on the particular portion of the earth immediately beneath it, so that at other places the potential-gradient should be zero. It is of little use to invoke the aid of the wind for the transportation of the positively charged air to regions where the replenishing action is not taking place, for theory shows that, owing to the conductivity of the air, this charge would disappear with great rapidity when the air carrying it had passed out of the region of replenishment.[9] In fact, the only way of

[9] W. F. G. Swann: "On the Origin and Maintenance of the Earth's Charge." *Terr. Mag.*, vol. xx. pp. 105–126, 1915.

accounting, even in part, for a distribution of potential-gradient over the earth's surface as a result of actions occurring at isolated places or periods, is to fall back upon the assumption of a very highly conducting layer in the upper atmosphere, as I explained when discussing C. T. R. Wilson's condensation theory.

The two most striking phenomena of atmospheric electricity requiring an explanation are the existence of the "Penetrating Radiation," and the origin of the earth's charge. It would be a distinct gain if we could reduce these two problems to one; if, for example, we could show that, whatever may be the explanation of the "Penetrating Radiation," provided only that we are willing to take its existence as an experimental fact, the maintenance of the earth's charge must follow as a necessary consequence. I think that there is perhaps some hope in this direction along lines which I shall endeavor to explain.

I must first recall to you one or two facts in relation to the ionization produced by γ rays. It will be recalled that ionization consists in the expulsion of an electron from a neutral atom by the ionizing agency. Now we know as an experimental fact that when γ rays ionize, the electrons which are emitted are shot off almost entirely in the direction of the incident γ rays, at least this is so for atoms of comparatively small atomic weight. Thus, for example, to quote some results by W. H. Bragg, it is found that if γ rays are allowed to fall upon a plate of carbon, 95 per cent. of the emitted electrons come off from the side of the plate at which the γ rays leave the carbon. The more penetrating the rays, the more pronounced the effect; further, the more penetrating the rays, the greater the speed and penetrating power of the electrons which are shot off. Now remembering these facts, what sort of action may we expect to be exhibited by the penetrating radiation which comes from above, supposing it to partake of γ ray properties? We may expect that when the penetrating radiation emits an electron from an atom of air, in the process of ionizing, that electron will be emitted almost entirely in a downward direction. Further, the great penetrating power of the penetrating radiation will insure that the electrons emitted will travel considerable distances in the atmosphere before they come to rest. Thus, in each layer of air, we shall have emissions of electrons in a downward direction, by the penetrating radiation from above, and absorption of electrons which have

been emitted by the penetrating radiation from the layers of air above. The earth's surface will receive electrons from the layers of air which are, so to speak, in striking distance of it, and it will consequently acquire a charge. As this charge grows, an electric field will be set up, and the charge will increase until this field has attained a value such that, owing to the conductivity of the atmosphere, as much negative electricity is returned to the various layers of air as is shot out of them by the action of the penetrating radiation. At each point of the atmosphere, there will consequently be a downward current of electrons which have been given high velocities by the penetrating radiation; this we shall refer to as the corpuscular current; and, there will be an ordinary conduction current which, while it involves both positive and negative ions (moving in opposite directions), has the effect of transferring the negative electricity back to the atmosphere. When a condition of equilibrium has been attained, the corpuscular current density must balance the conduction current density, and it will be interesting, from this standpoint, to submit the theory to examination as regards order of magnitude.

Taking for the purpose of illustration a simplified case where the penetrating radiation considered is all directed vertically downwards, if q is the number of electrons liberated per c.c. per second by the penetrating radiation, and h the average distance which an electron travels from its point of origin, in the original direction of its motion, the corpuscular current density will be qeh, so that if i is the conduction current density, we should have:

$$i = qeh$$

where e is the electronic charge. If we take account of the fact that the penetrating radiation comes from all directions contained within a hemisphere, the effect is merely to introduce a factor 0.5 on the right-hand side.

Suppose now we put for q, the value 3, which is approximately equal to the portion of the ionization per c.c. per second in a closed vessel, which is due to the radiation from above. Then putting $i = 6.5 \times 10^{-7}$ electrostatic unit, which is the average value from a large number of observations, and $e = 4.8 \times 10^{-10}$, we find that h must be 9 metres.

Now 9 metres is by no means a large range of path to assume for electrons which are emitted by a radiation as penetrating as

the "Penetrating Radiation"; for Eve has found β rays from radioactive substances having ranges as large as 7 metres.

The theory is attended by one difficulty which is not, I think, vital, however. We know that electrons which are emitted from atoms with high speed will themselves ionize a gas through which they pass, so that, ought we not to conclude that the very high-energy electrons which are emitted by the penetrating radiation would themselves produce many more ions in the atmosphere? Ought we not to conclude, in fact, that the three ions per c.c. per second which we have taken as the basis of our calculations represent the net ionization produced by the ejection of the original electrons and the ionization which these electrons themselves produce subsequently. An ordinary β ray produces about 40 ions per centimetre of its path, so that, in its whole journey in the atmosphere, it produces a very large number of ions. If we adopt this view as to the ionization by the electrons originally ejected by the penetrating radiation then, quite apart from all questions of the origin of the earth's charge, our views as to the meaning of the measurements made in closed vessels will require very careful scrutiny. A close examination of the matter shows, however, that our primary difficulty may not be as great as at first sight appears to be the case. Although the velocity of light represents the greatest velocity which an electron can have, and ordinary β rays have velocities 95 per cent. and more of this velocity, we must not too readily assume that there is practically no difference between the behavior of an electron with velocity, say, 98 per cent. of the velocity of light, and one with velocity 99.8 per cent. of that velocity for example. For theory shows that certain properties which are characteristic of the fields of high-speed corpuscles only commence to make their appearance when the velocity attains a value very near to that of light. With electrons of moderate β ray velocity there is, of course, no question as to the fact that ionization does occur all along the path; but, though the considerations involved are too complex for brief presentation, it may be remarked that the theoretical properties of electrons of very high energy are such as to suggest that they may not fritter away their energy in ionization as readily as do those with moderate β ray energy. It may be that when an electron is emitted from a molecule by a radiation as hard as the penetrating radiation, it travels through the air without ionizing the molecules, and without, in

general, suffering any appreciable change in its energy; but that if the conditions of encounter with a molecule are favorable, it becomes completely absorbed by the molecule, and in so doing, sends out a radiation of a type similar to that which caused its emission from the parent atom. To those who are familiar with modern developments along the line of the "Quantum Theory," a view of this kind will probably appear much less visionary than it would do on the basis of our earlier views.

It may be remarked that the difficulty I have discussed, with regard to the question of ionization by the high-energy electrons, is one which would be felt in any theory in which such electrons are invoked. It would appear, for example, in the case of Simpson's tentative suggestion as to the maintenance of the earth's charge by corpuscles emitted from the sun.

THE ORIGIN OF THUNDERSTORMS.

It would be easy to devote the whole of this lecture to the subject of thunderstorms, so the treatment to which I shall confine myself must necessarily be brief. The theory which has been most successful in explaining the facts is that suggested by G. C. Simpson. The theory has also received considerable development on the meteorological side in the writings of W. J. Humphreys in the JOURNAL of the Institute.

The theory is founded on the experimental fact that if water is broken into drops by allowing it to fall upon a rising column of air, the drops are found to be positively charged while the air receives a negative charge. Now experiment has shown that it is impossible for drops of water to fall through air which is rising with a velocity greater than 8 metres per second. If the drops are very small they will be blown up by a rising column of air, and it is in general necessary for a drop to have a certain minimum mass before it can fall through a column of air rising with specified velocity. If the velocity is as high as 8 metres per second, however, the size which the drop would have to attain in order to fall through the column would be so great that the drop would be broken up by the air stream, even if it succeeded in attaining the necessary size temporarily. The smaller drops formed as a result of the disintegration would, of course, be carried upwards by the air stream.

Now it is a matter of common experience that thunderstorms are always preceded by high winds, and a close examination of

the phenomena shows that, in the storms which give rise to electrical discharges, columns of air are to be found rising with very considerable velocity. We may liken one of these columns of air to an hour-glass, or dice box. At the bottom the column is wide, and the velocity is small. At the narrowest part of the column the air attains its maximum velocity, while at the top it fans out and the velocity again becomes small. Now in the period prior to a thunderstorm, the air is very humid. As this air rises and becomes cooled in the process, it eventually reaches a temperature at which drops of water begin to condense out. At first, these drops are carried upwards with the stream; but, as they grow, they eventually reach a size at which they start to fall. Suppose that when this occurs, the drops are some distance above the narrowest part of the air column. Then, as they fall and continue to grow in size they eventually reach a place where the velocity is sufficiently great to break them up. The smaller drops become positively charged in the process and the air receives a negative charge. The small drops now start their ascent again, although, of course, with velocity less than that of the air. As they rise they grow, and eventually the whole process is repeated again. The cycle may be gone through several times, the drops becoming charged more and more each time. The free negative ions which ascend with the rising air eventually coalesce with the finer water drops to be found at the top of the cloud, so that we eventually attain a state of affairs in which the water at the top of the cloud is highly charged negatively, while that in the middle of the cloud is highly charged positively. When the accumulations of charges are sufficient to result in a field which will break down the insulation of the air at some point, a lightning flash follows. It thus appears that the high winds which are associated with the thunderstorm are in no sense a result of the electrical manifestations; the electrical phenomena are themselves secondary features resulting from the air motion.

During the turbulence associated with the storm, some of the large drops of positively charged rain which have gone through the cycle of breaking and reformation several times get carried to places where the velocity of the upward current is not sufficient to break them up, and they fall to earth. In this way we find the explanation of the experimental fact that the heavy rain of the thunderstorm is found to be positively charged. On the other hand, the rain which accumulates at the top of the cloud, and

which is negatively charged is of the finer type, and may be expected to fall to earth only in the lulls between the periods of most violent activity, or at places somewhat remote from the storm centre. Here, again, experiment supports the conclusions in showing that the finer rain which falls during the quiet periods of the storm is negatively charged.

Of fire-balls, thunderbolts and the like, time forbids me to speak. These very interesting phenomena have been discussed very clearly in Professor Humphrey's articles, "The Thunderstorm and Its Phenomena,"[10] published in the JOURNAL of this Institute; and, it will suffice to say that, in spite of the fantastic behavior which is sometimes attributed to these more rare attendants of the thunderstorm, there does not appear to be anything which is not capable of explanation in terms of our modern views of electrical phenomena.

From time immemorial, the thunderstorm has been the emblem of all that is gigantic, mysterious, and awe-inspiring in the manifestations of nature's power; and, there is perhaps an element of poetic justice in the fact that this monster's secrets have been among the first of the greater problems of atmospheric electricity to be satisfactorily unveiled. Cosmical physics as a whole has gained less from the fruits of modern advances than have the more purely laboratory sciences. The discovery of the electron, and the development of the electrical theory of matter have indeed revolutionized our notions of how things happen in molecular physics; and atmospheric electricity has reaped as much advantage from the work of the past thirty years as perhaps any other branch of cosmical physics. Yet, when we think of the great detail in which modern theory coördinates most of the more intricate and less obvious facts of chemistry and physics, it is rather staggering to recall how many problems of a cosmical nature are yet far from a satisfactory explanation. How long will gravitation and the origin of the earth's magnetic field continue to baffle all who ponder over nature's mysteries! How long will the great science of meteorology, including atmospheric electricity continue to hold back some of its most striking secrets! And, when these have succumbed to the onward progress of science, when, if ever, shall we hope for that great consummation of man's scientific effort—the coördination into the general scheme of things of the greatest puzzle of them all, the mystery of organic life!

[10] JOURNAL OF THE FRANKLIN INSTITUTE, November and December, 1914.

OPTICS OF THE AIR.*

BY

W. J. HUMPHREYS.

Professor of Meteorological Physics, United States Weather Bureau.

CHAPTER IV.

REFRACTION PHENOMENA: REFRACTION BY ICE CRYSTALS.

Introduction.—The cirrus clouds and others formed at temperatures considerably below 0° C. usually consist of small but relatively thick snowflakes with flat bases, or ice spicules with flat or, rarely, pyramidal bases, always hexagonal in pattern and detail, as shown by Fig. 151 from Bentley's remarkable collection of photomicrographs of snow crystals.

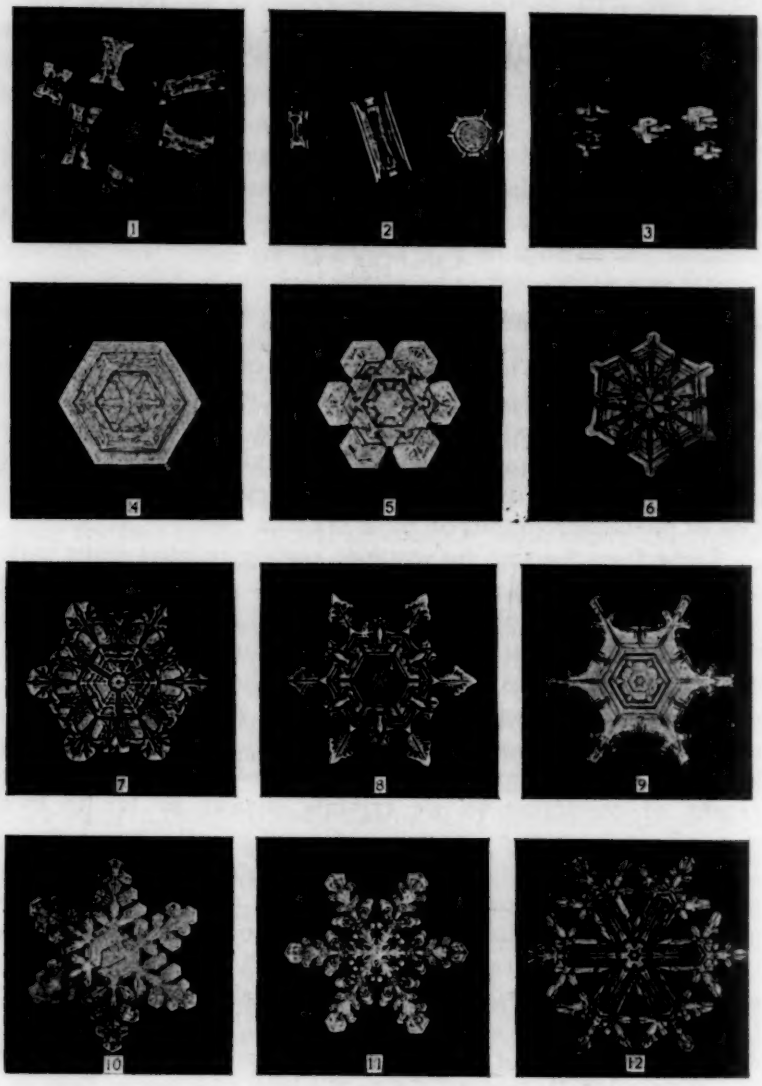
Light from the sun, for instance, obviously takes many paths through such crystals and produces in each case a corresponding and peculiar optical phenomenon. Several of these phenomena, the halo of 22° radius, the halo of 46° radius, the circumzenithal arc, parhelia, etc., are quite familiar and their explanations definitely known. Others, however, have so rarely been seen and measured that the theories of their formation are still somewhat in doubt. Finally, many phenomena, theoretically possible, as results of refraction by ice crystals, appear so far to have escaped notice.

Prismatic Refraction.—Since the phenomena caused by the passage of light through ice crystals are numerous, it will be most convenient, in discussing them, first to obtain general equations for prismatic refraction, and then to substitute in these equations the numerical constants applicable to each particular case.

Deviation.—Let A (Fig. 152) be the angle between two adjacent faces of a prism; let CE be the path of a ray of light in a plane normal to their intersection (direction of travel imma-

* Continued from page 488, Vol. 188, October, 1919.

FIG. 151.



Snow crystals (Bentley).

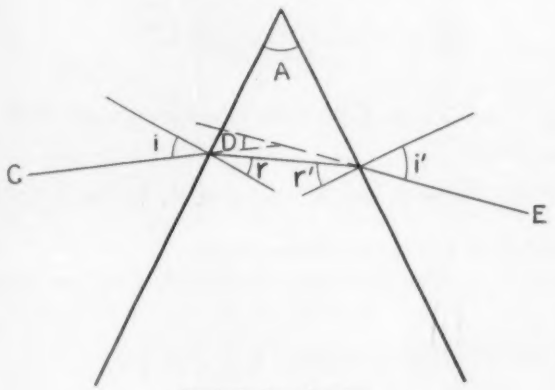
terial); let i and i' be the angles of incidence and r and r' the corresponding angles of refraction. Then the change in direction, D, of the ray is given by the equation,

$$D = i + i' - (r + r') = i + i' - A \ldots\ldots\ldots\ldots (1)$$

Minimum Deviation.—Minimum deviation occurs when $\frac{dD}{di} = 0$ and $\frac{d^2D}{di^2} > 0$ But when

$$\frac{dD}{di} = 1 + \frac{di'}{di} = 0, \; di = -di'.$$

FIG. 152.



Deviation by refraction.

and as

$$r + r' = A, \quad dr = -dr'.$$

Also, from the law of refraction,

$$\sin i = \mu \sin r.$$
$$\sin i' = \mu \sin r'$$

Hence, directly,

$$\frac{\sin i}{\sin i'} = \frac{\sin r}{\sin r'},$$

or

$$\sin i \sin r' = \sin i' \sin r$$

and, by differentiation, if $di = -di'$, and $dr = -dr'$,

$$\frac{\cos i}{\cos i'} = \frac{\cos r}{\cos r'}$$

or

$$\cos i \cos r' = \cos i' \cos r.$$

By addition, $\cos(i-r)=\cos(i'-r)$, or $i-r=i'-r$.
By subtraction, $\cos(i+r')=\cos(i'+r)$, or $i+r'=i'+r$.

Hence, if, as assumed, $\frac{dD}{di}=0$,

$$i = i', \text{ and } r = r' = \frac{A}{2}$$

From

$$\frac{dD}{di} = 1 + \frac{di'}{di} = 1 + \frac{\frac{\mu \cos r' \, dr'}{\cos i'}}{\frac{\mu \cos r \, dr}{\cos i}} = 1 - \frac{\cos i \cos r'}{\cos i' \cos r},$$

it follows, by a little reduction, that when $\frac{dD}{di}=0$

$$\frac{d^2D}{di^2} = \frac{\mu \cos^2 r \sin i' - \cos^2 i \sin r}{\mu \cos i \cos^2 r}.$$

But

$$\mu > 0, \cos^2 r > \cos^2 i, \sin i > \sin r, \text{ and } \mu \cos i \cos^2 r > 0.$$

Hence, when

$$\frac{dD}{di} = 0, \text{ that is, when } r = r', \frac{d^2D}{di^2} > 0$$

and the deviation has its minimum value.

Writing D_0 for the minimum deviation, it follows that

$$D_0 = 2i - A. \quad \text{(2)}$$

Hence, from $\sin i = \mu \sin r$, and $r = \frac{A}{2}$, we get

$$\sin \frac{D_0 + A}{2} = \mu \sin \frac{A}{2} \quad \text{(3)}$$

Maximum deviation, D_m, obviously occurs when

$$i = 90°,$$

or, since μ for ice = 1.31, when $r = 49° 46'$.
Since

$$D = i + i' - A$$
$$D_m = 90° + i' - A$$

and

$$\sin(D_m + A - 90°) = \mu \sin r' = \mu \sin(A - 49° 46')$$

or

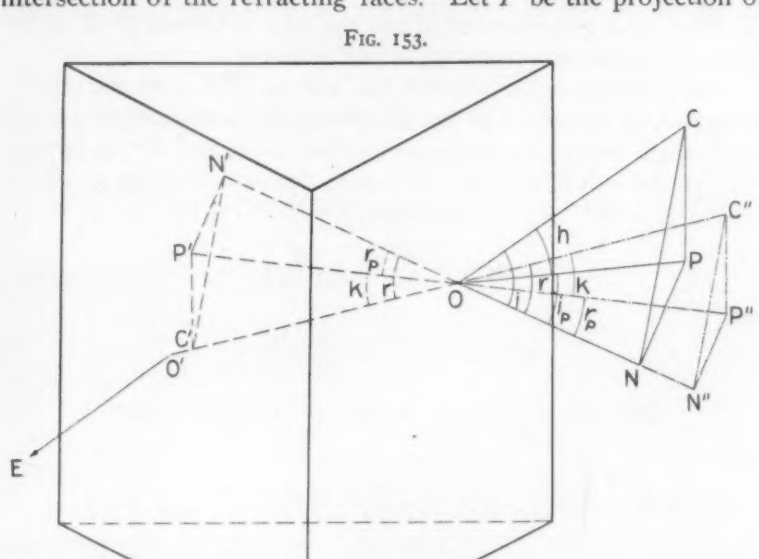
$$\cos[(180° - A) - D_m] = \mu \sin(A - 49° 46') \quad \text{(4)}$$

The above equations apply only to refraction in a plane normal to the intersection of the faces of the prism. When the incident ray is inclined to this plane, the effective angle of refraction is increased, and as such inclination usually occurs in the case

of floating ice crystals it is necessary, in the study of halos, to evaluate its effect on the deviation.

Let the ray, CE (Fig. 153), enter a prism at O and leave it at O'. Let ON be the normal to the prism face at O, and let OP be the projection of CO onto a plane through O normal to the intersection of the refracting faces. Let P be the projection of

FIG. 153.



Refraction of rays inclined to principal plane.

C, and let the plane CNP be normal to ON. Similarly, let P' be the projection of C' on the principal plane, PON, and let the plane $C'P'N'$ be perpendicular to the normal ON extended; then, since the refracted ray OC' lies in the plane OCN, it follows that the triangles CNP and $C'N'P'$ are similar.

Therefore,

$$\frac{\sin h}{\sin k} = \frac{\sin i}{\sin r},$$

and

$$\sin h = \mu \sin k,$$

in which h and k are the angles between the ray and principal plane before and after refraction, respectively.

Similarly, if h' and $k' = k$ are the angles at O' corresponding to h and k at O,

$$\sin h' = \mu \sin k' = \mu \sin k.$$

Hence,

$$h = h'$$

That is, the inclination of the ray to a principal plane is the same after leaving a prism as before entering it.

Let i_p be the angle between the normal, ON, and the projection, OP, of the ray, CO, on the principal plane through O, and r_p the angle betwen the normal extended, ON', and the projection, OP' of the refracted ray, OC', onto the principal plane. Then (perhaps better seen by reversing $OC'N'P'$ to $OC''N''P''$),

$$\sin i_p = \frac{NP}{OP} = \frac{NP}{OC \cos h},$$

and

$$\sin r_p = \frac{N'' P''}{O P''} = \frac{N'' P''}{OC'' \cos k}.$$

$$\text{But } NP = \frac{OC}{OC''} \mu N'' P'' = \mu N'' P'' \text{ if } OC = OC'',$$

and

$$\sin i_p = \mu \frac{\cos k}{\cos h} \sin r_p.$$

Hence, rays inclined to the principal plane of a prism of refractive index μ are bent as if in the principal plane of a similar prism of index, μ', where

$$\mu' = \mu \frac{\cos k}{\cos h} = (\mu^2 - \sin^2 h)^{\frac{1}{2}} (1 - \sin^2 h)^{-\frac{1}{2}} [188]$$

The minimum deviation, therefore, D'_0, measured in or projected onto the principal plane, of such rays is given by the equation,

$$\sin \frac{D'_0 + A}{2} = \mu \frac{\cos k}{\cos h} \sin \frac{A}{2}, \quad \ldots\ldots\ldots\ldots (5)$$

and the maximum, D'_m, by the equation,

$$\cos [(180° - A) - D'_m] = \frac{\cos k}{\cos h} \sin (A - a), \quad \ldots\ldots (6)$$

[188] It may be interesting to note that this relation between the inclination of a ray to the principal plane of a prism and its deviation by that prism explains the curvature of spectrum lines as seen in an ordinary straight slit prism spectroscope.

in which α is the limiting value of the angle of refraction for the index μ', when

$$\mu' = \mu \frac{\cos k}{\cos h}.$$

The largest or limiting value of h at which light can still pass through the prism obviously is determined by the equation,

$$\frac{D + A}{2} = 90°,$$

in which D is the minimum deviation as projected on the principal plane.

In this case

$$\sin \frac{D + A}{2} = \mu \frac{\cos k}{\cos h} \sin \frac{A}{2} = 1. \quad \text{(7)}$$

Therefore,

$$\frac{1}{\sin^2 \frac{A}{2}} = \mu^2 \frac{(1 - \sin^2 k)}{\cos^2 h} = \frac{\mu^2 - \sin^2 h}{\cos^2 h}$$

and

$$\cos h = \sqrt{\mu^2 - 1} \tan \frac{A}{2}.$$

Hence, when $A = 60°$, as between alternate sides of a hexagonal ice prism, or snowflake, the limiting value of h for $\mu = 1.31$, is 60° 45′, and when A is 90°, as between base and a side, 32° 12′.

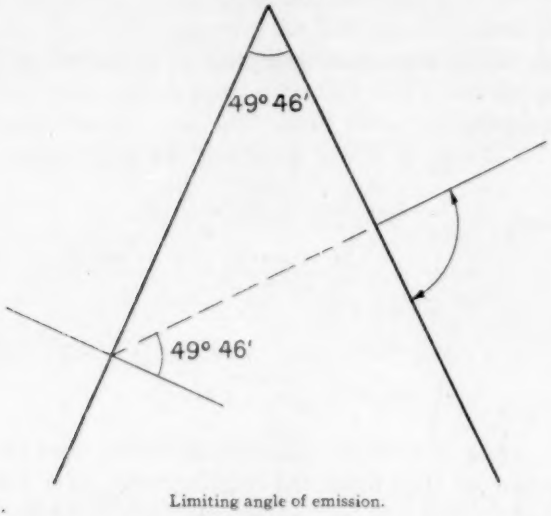
Internal Total Reflection and Its Effect on the Passage of Light Through Ice Crystals.—Since the limiting value of the "angle of incidence" is 90°, and the refractive index of ice 1.31, it follows that total reflection of an internal ray occurs at the angle α, given by the equation,

$$\sin 90° = 1.31 \sin \alpha = 1.31 \sin 49° 46'.$$

An internal ray, therefore, cannot leave an ice crystal if the angle it makes with the normal is greater than 49° 46′. Hence, as is clear from Fig. 154, a ray of light in the principal plane, and also most rays out of it, will pass through an ice crystal between faces whose inclination is not greater than 49° 46′ at all angles of incidence (measured on the base side of the normal) from 0° to 90°. On the other hand, no light can pass through an ice crystal at any angle of incidence between planes whose inclination is greater than $2 \times 49° 46'$. In proof of this latter

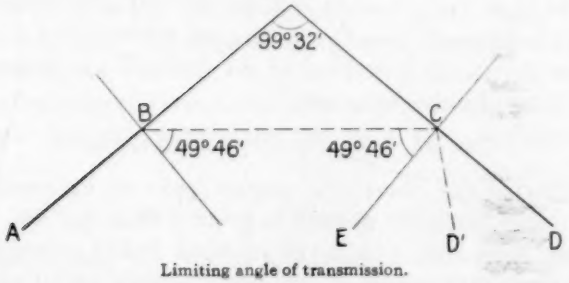
statement, let AB (Fig. 155) be a ray grazing the side of a crystal whose angle of refraction is 99° 32′ and entering at B. It will reach the opposite face at C, and either pass out in the direction CD or else suffer total reflection. But as CD lies along the face of the crystal, it is clear that any decrease of the angle of incidence at B from 90°, or increase of the inclination of the

FIG. 154.



Limiting angle of emission.

FIG. 155.



Limiting angle of transmission.

crystal faces to each other, each of which increases the angle BCE, causes total reflection at C, and thereby prevents transmission. Refracting angles intermediate between the above extremes obviously transmit light incident through an angular range less than 90° and greater than 0°.

The largest angle of incidence clearly is 90°, and the smallest i, as determined by the equation,

$$\sin i = \mu \sin r = 1.31 \sin (A - 49° 26')$$

If, then, $A = 60°$, $i = 13° 54'$, giving a transmission range of 76° 6′; if $A = 90°$, $i = 58° 25'$, range 31° 35′; and similarly for other possible values of A.

General Illumination of the Sky Through Ice Crystals.—The deviation of a ray of light through refraction and n internal reflections obviously is given by the equation,

$$D = i + i' + n\pi - \Sigma A,$$

in which ΣA is the sum of the several angles passed by the ray in its course through the crystal. If these angles are all equal the equation becomes

$$D = i + i' + n\pi - (n + 1) A.$$

If, then, as frequently is the case, the ice crystal is a thick hexagonal disk floating horizontally, the position it oscillates about in falling, both angles passed by a once reflected ray will be 90°, provided the entering and emergent branches lie in the same plane, and the deviation will be

$$D = 2 i,$$

as readily seen in Fig. 156.

Hence, such crystals illuminate the sky at all distances from the sun out to 116° 50′. The effect, however, is not sufficiently striking ordinarily to arrest attention.

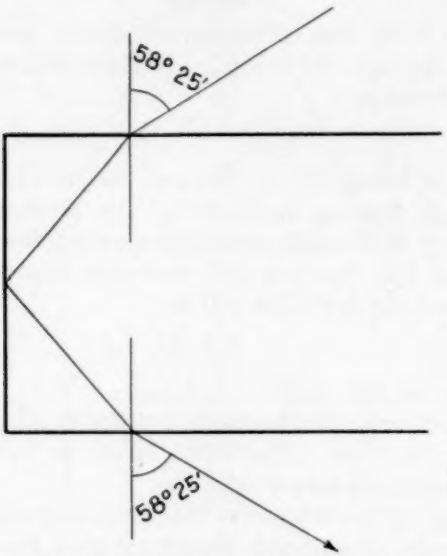
Parhelia of 22°.—Whenever the air through any depth or at any level contains innumerable hexagonal snow crystals with their sides vertical (the position about which relatively broad crystals oscillate) two colored bright spots, known as parhelia or sun dogs, appear at 22°, or more, from the sun, one to the right, the other to the left. Each bright spot obviously is in the direction of maximum light and, therefore, the direction of minimum refraction. When the refraction is in a principal plane, that is, as the sides of the crystal are vertical, when the sun is on the horizon, the angular distance, D_0, of each spot, also on the horizon, is given by the equation, derived from (3),

$$\sin \frac{D_0 + 60°}{2} = \mu \sin \frac{60°}{2}$$

For yellow light ($\mu = 1.31$), $D_0 = 21^\circ\ 50'$; for red light ($\mu = 1.307$), D_0 $21^\circ\ 34'$; and for violet ($\mu = 1.3117$), $D_0 = 22^\circ\ 22'$. The order of the colors, therefore, counting from the sun, is red, yellow, etc., to violet, and the length or dispersion $48'$ for a point source. For the sun, diameter $30'$, the total length is $1^\circ\ 18'$, and width $30'$.

Since the above are minimum angles, it follows that slight changes in either the inclination or orientation of the crystals causes rays of each color to come also from somewhat greater

FIG. 156.



Illumination of the sky by flat snow crystals.

distances from the sun. Hence, the only color thus produced that appears approximately pure is the darker portion of the red. Yellow and green are also moderately distinct, but blue, and especially violet, scarcely perceptible because of so much admixture of colors.

When the angular elevation of the sun is h, the distance, D', in azimuth, of each of these parhelia from the sun is given by the equation, derived from (5),

$$\sin \frac{D' + 60^\circ}{2} = \mu \frac{\cos k}{\cos h} \sin \frac{60^\circ}{2}.$$

The angular distance, Δ_0, measured on the arc of a great circle, between the sun and each of these parhelia, may be found from the right spherical triangle formed by the three sides: zenith to sun (compliment of h); zenith to mid point between sun and a parhelian; "mid point" to sun, $\frac{\Delta_0}{2}$. The angle thus formed at the zenith is $\frac{D'}{2}$, and the angle at the "mid point" 90°. Hence,

$$\sin \frac{\Delta_0}{2} = \cos h \sin \frac{D'}{2}$$

For $\mu = 1.31$ the following relations [187] exist between h, D', and Δ_0.

h	D'	Δ_0
0°	$D_0 = \Delta_0 =$	21° 50′
5°	22° 2′	21° 56′
10°	22° 30′	22° 10′
15°	23° 20′	22° 32′
20°	24° 34′	23° 4′
25°	26° 22′	23° 50′
30°	28° 44′	24° 49′
35°	31° 56′	26° 3′
40°	36° 20′	27° 38′
45°	42° 30′	29° 42′
50°	51° 30′	32° 26′
55°	66° 2′	36° 26′
60°	98° 48′	44° 38′
60°45′	120° 0′	50° 4′

All the above values pertain, as explained, to minimum deviation. But as the orientation of the crystals is fortuitous, it follows that all possible deviations from minimum to maximum will occur, and the parhelia, therefore, be drawn out into streaks the lengths of which depend upon their angular altitude.

The maximum deviation for refraction in a principal plane (for sun on the horizon when the crystal edges are vertical) is given by the equation, derived from (4),

$$\cos (180° - 60° - D_m) = \mu \sin (60° - 49° 26').$$

The value of the maximum deviation in azimuth, D_m, corresponding to the solar altitude, h, is given by equation (6), and the actual maximum, Δ_m, by the equation,

$$\sin \frac{\Delta_m}{2} = \cos h \sin \frac{D'_m}{2}.$$

[187] Pernter-Exner, *Meteorologische Optik*, pp. 314, 315.

The following table [188] gives interesting relations between the quantities indicated:

h	D_m	Δ_m	$\Delta_m - \Delta_0$
0°	43° 28′	43° 28′	21° 38′
5°	43° 38′	43° 26′	
10°	44° 8′	43° 24′	21° 16′
15°	44° 59′	43° 20′	
20°	46° 15′	43° 18′	20° 18′
25°	48° 0′	43° 16′	
30°	50° 17′	43° 10′	18° 22′
35°	53° 15′	43° 4′	
40°	57° 9′	42° 58′	15° 22′
45°	62° 28′	43° 2′	
50°	68° 48′	43° 10′	10° 46′
55°	81° 3′	43° 44′	
60°	104° 54′	46° 44′	2° 6′
60°45′	120° 0′	50° 4′	0° 0′

From the computed values of $\Delta_m - \Delta_0$, fully supported by observations, it appears that when the angular elevation of a parhelion of 22° is moderate to small, 20° or less, it may extend over an arc, parallel to the horizon, of more than 20°. The end next the sun, produced by minimum deviation, is colored, beginning with red, through a short range. Similarly, the distal end, due to maximum deviation, is also colored, terminating with violet, though always too faint, perhaps, to be distinctly seen. Through the rest of its length the blending of the colors is quite complete, giving white, of course, as the result.

At greater altitudes the possible lengths of the parhelia of 22° become less and less, as shown by the table, though the color distribution remains the same.

Halo of 22°.—When the refracting edges of the ice crystals are vertical, as they tend to be in the case of relatively thin snowflakes falling through still air, parhelia are produced, as just explained. But, in general, these edges lie in all directions, especially at the windy cirrus level and when the crystals are of the short columnar type; and as refracted light reaches an observer in every plane through his eye and the sun (or moon) to which the refracting edges are approximately normal, it follows that the effect produced by fortuitously directed snow crystals must be more or less symmetrically distributed on all sides of the exciting luminary. There may, however, be a maximum brightness both

[188] Pernter-Exner, *Meteorologische Optik*, p. 317.

directly above and directly below the sun since ice needles tend to settle with their refracting edges horizontal.

As before, when the refracting angle is 60° and $\mu = 1.31$, corresponding to yellow light, $D_0 = 21° 50'$, and is independent of solar elevation. The inner portion of this, the most frequent and best known of all halos, is red, because light of that color is least refracted. Other colors follow, with increase of distance, in the regular spectral sequence, but with decrease of wave-length they so rapidly fade that even green is indistinct and blue seldom detected. This is owing to the variation in deviation caused by the tipping of the needles, as previously fully explained.

The brightest portion of the ring clearly is at the angle of minimum refraction from the sun. With increase of distance, light produced in this manner gradually fades (not all the crystals are ever simultaneously in position to give minimum refraction) until it ceases to be perceptible at 15° to 20° beyond the inner portion, or, say, 40° from the sun. On the other hand, no such light reaches the observer from places within the ring of maximum brightness, and, therefore, this portion of the sky is comparatively dark, except, and for an entirely different reason, to be explained later,[189] near the sun itself.

When the sun is within 10° of the horizon, the halo of 22°, and the parhelia of 22°, are practically superimposed. At greater altitudes they become distinctly separated, as per the accompanying table [190] for $\mu = 1.31$, in which h = solar elevation, Δ_0 = parhelic angular distance from the sun and D_0 = angular distance of halo from sun.

h	$\Delta_0 - D_0$
0°	0° 0′
10°	0° 20′
20°	1° 14′
30°	2° 59′
40°	5° 48′
50°	10° 36′
60°	22° 48′
60° 45′	28° 14′

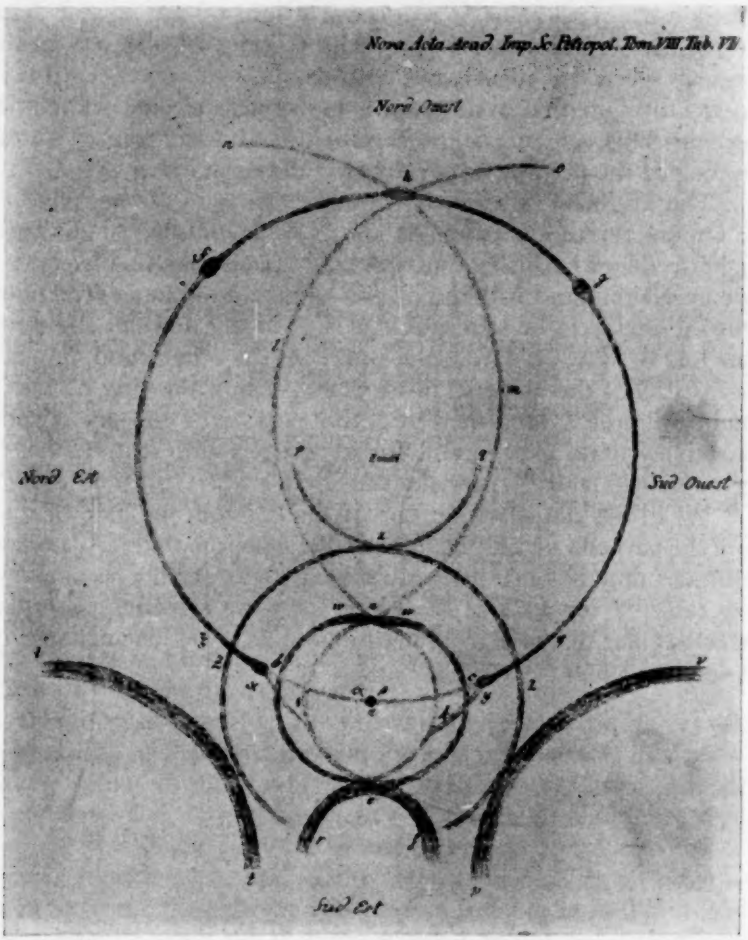
Arcs of Lowitz, or Vertical Arcs of the 22° Parhelia.—On rare occasions oblique extensions of the parhelia of 22°, concave towards the sun and with red inner borders, are seen, in addition to their horizontal tails, above described. These are known

[189] See chapter on diffraction.

[190] Pernter-Exner, *Meteorologische Optik*, p. 321.

as the arcs of Lowitz, after the astronomer who described them [191] as seen in the famous Petersburg halo complex (Fig. 157) of July 18, 1794. Their general explanation is simple, though exact

FIG. 157.



Petersburg halo of July 18, 1794.

computations of their outlines and of the relative intensities of their parts for different altitudes of the sun are rather tedious.

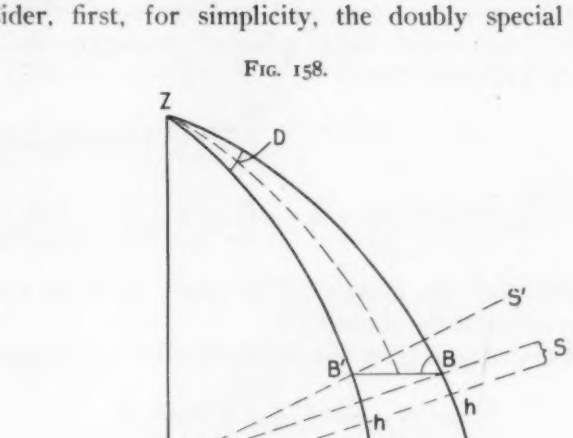
[191] *Nova acta Acad. Petropol.*, 8, 1794, p. 384.

As already explained, parhelia and their horizontal extensions are produced by ice crystals whose principal axes are vertical, the former by those set to minimum refraction, and the latter by crystals turned more or less from this unique position.

When, however, the principal axes oscillate about the vertical, as they obviously do in the case of snowflakes, the arcs of Lowitz, or obliquely vertical extensions of the parhelia of 22°, necessarily are produced, though rarely seen, because of the diffusion of their light in the midst of a general glare, as will be explained below.

Consider, first, for simplicity, the doubly special case in

FIG. 158.



Formation of the arc of Lowitz, crystals vibrating in solar vertical.

which the sun is on the horizon and the principal axes of the crystal oscillate in a vertical plane passing through the sun. Let C (Fig. 158) be the position of an ice crystal whose principal axis is in the direction CZ. Let an observer be at O and let the incident ray, SC, lying between Z and P make the angle h with the principal plane, CP. On emerging this ray, in its new direction, $S'O$, has the same inclination as formerly to the axis. Hence, SC and $S'C$ may be regarded as elements of a right cone of vertex C and axis CZ, and as the plane CZS is vertical, if S is on the horizon, as seen from O, the element CS (lowest element), being parallel to OS, owing to the great distance of S, will lie in

a plane tangent to the cone and parallel to the plane of the horizon, while every other element, such as CS', will lie above it. S', therefore, the apparent position of S due to refraction of the ray SC by the ice crystal at C is above this plane and, also, as seen from O at the same angular distance above the horizon. Similarly, when the axis of the crystal is tipped beyond the vertical in the opposite direction, S' drops below the horizon.

Further, let ZP and ZP' be arcs of great circles intersecting SC and $S'C$ at B and B', respectively. Then the projection of the deviation SCS', or SOS', on the principal plane is given by the angle D between these arcs, and on bisecting D, thus dividing ZBB' into two equal right spherical triangles, and putting $BB' = \Delta$, it is obvious that

$$\sin\frac{\Delta}{2} = \sin\frac{D}{2}\cos h \text{ - - - - - - - - - - - - - - (1)}$$

and

$$\cot B = \tan\frac{D}{2}\sin h \text{ - - - - - - - - - - - - - (2)}$$

Clearly, then, the locus of S' is given when the arc BB' is known in terms of the angle B.

On eliminating h from the above equations, it appears that

$$\cot^2 B = \tan^2\frac{D}{2}\cos^2\frac{\Delta}{2} - \sin^2\frac{\Delta}{2} \text{ - - - - - - - - - - (3)}$$

Hence, that either Δ or B may be found when the other is given, it remains only to express $\tan^2\frac{D}{2}$ in terms of a function or functions of Δ.

But from (7), (p. 613),

$$\sin^2\frac{D+A}{2} = \mu^2\frac{\cos^2 k}{\cos^2 h}\sin^2\frac{A}{2}$$

$$= \left\{1 + (\mu^2 - 1)\sec^2 h\right\}\sin^2\frac{A}{2}.$$

Also from (3), (p. 610),

$$\mu^2 = \frac{\sin^2\frac{D_0 + A}{2}}{\sin^2\frac{A}{2}}$$

and from (1),

$$\sec^2 h = \frac{\sin^2 \frac{D}{2}}{\sin^2 \frac{\Delta}{2}}.$$

Hence,

$$\frac{\sin^2 \frac{D+A}{2} - \sin^2 \frac{A}{2}}{\sin^2 \frac{A}{2}} = \frac{\sin^2 \frac{D_0+A}{2} - \sin^2 \frac{A}{2}}{\sin^2 \frac{A}{2}} \times \frac{\sin^2 \frac{D}{2}}{\sin^2 \frac{\Delta}{2}}$$

or

$$\sin^2 \frac{\Delta}{2} \sin\left(\frac{D}{2} + A\right) \sin \frac{D}{2} = \sin^2 \frac{D}{2} \sin\left(\frac{D_0}{2} + A\right) \sin \frac{D^2}{2}.$$

Dividing by $\cos \frac{D}{2} \cos A$,

$$\sin^2 \frac{\Delta}{2} \tan \frac{D}{2} + \sin^2 \frac{\Delta}{2} \tan A = \frac{\sin\left(\frac{D_0}{2} + A\right) \sin \frac{D_0}{2} \tan \frac{D}{2}}{\cos A}$$

$$= \frac{1}{2}\left\{1 - \frac{\cos(D_0 + A)}{\cos A} \tan \frac{D}{2}\right\}$$

Putting

$$\frac{\cos D_0 + A}{\cos A} = \cos\beta = \cos 73^\circ\ 30'$$

$$\tan \frac{D}{2} = \frac{2 \tan A \sin^2 \frac{\Delta}{2}}{\cos\Delta - \cos\beta} \quad \text{(4)}$$

On using this value of $\tan \frac{D}{2}$ (3) reduces to

$$\cot B = \frac{\sin \frac{\Delta}{2} \sqrt{4 \sin^2 \frac{\Delta}{2} \cos^2 \frac{\Delta}{2} \tan^2 A - (\cos\Delta - \cos\beta)^2}}{\cos\Delta - \cos\beta} \quad \text{(5)}$$

From (5) B is readily computed for any assumed value of Δ, as is also D form (4). Further, h can be found from (1) when Δ and D are known, or from (2) when B and D are known.

The following table, copied from Pernter-Exner,[192] as are

[192] *l. c.*, p. 327.

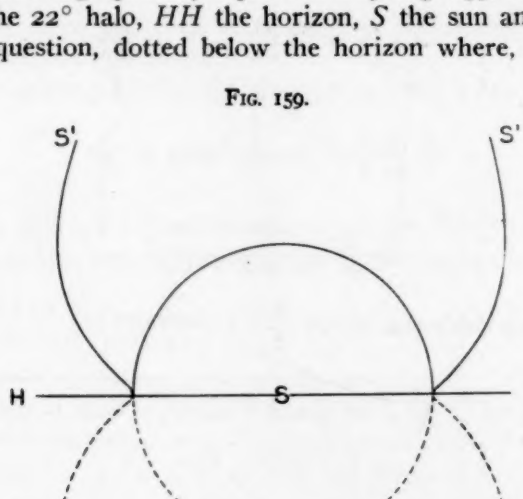
most of the above equations, gives data for drawing the locus of S' when the sun is on the horizon.

Since the value of Δ in this table increases with μ, other things being equal, it follows that these arcs must be colored and that their inner borders must be red, as stated.

h	Δ	B
0°	21° 50′	90° 0′
5°	21° 55′	89° 5′
10°	22° 10′	88° 1′
15°	22° 38′	86° 54′
20°	23° 04′	85° 45′
25°	23° 51′	84° 19′
30°	24° 49′	82° 38′
35°	26° 3′	80° 38′
40°	27° 38′	78° 5′
45°	29° 42′	74° 36′
50°	32° 26′	69° 43′
55°	36° 26′	61° 58′
60°	44° 38′	44° 41′
60° 45′	50° 4′	33° 29′

This table is graphically represented by Fig. 159, in which the circle is the 22° halo, HH the horizon, S the sun and $S'S'$ the curve in question, dotted below the horizon where, of course,

FIG. 159.

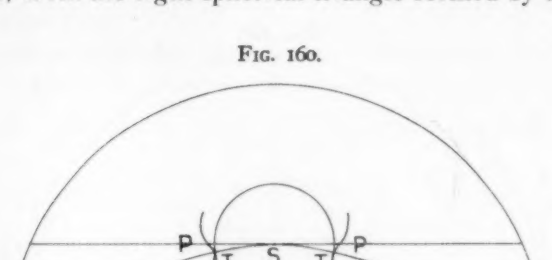


Arc of Lowitz, sun on horizon, crystals vibrating in solar vertical.

like the under portion of the circle, it is invisible, except from a suitable elevation.

Obviously, the optical effect is independent of the manner by which the inclination of the principal axis of the crystal to the incident ray is produced, and, therefore, crystals tilted in the vertical plane through the sun to an angle, $E + h$, to the plane of the horizon give the same result, if E is the solar elevation, as do crystals tilted to only the angle h in this plane when the sun is on the horizon. The points of contact with the halo of 22°, being due to crystals whose principal axes are normal to the incident rays, lie, therefore, at equal altitudes on opposite sides of the halo and in a plane that passes through the sun. Consequently, the angular altitude of these points is less than that of the sun, except when the latter is on the horizon. If, as before, E is the elevation of the sun and E' that of the points of contact in question, then, from the right spherical triangle formed by the radius

FIG. 160.



Arcs of Lowitz, elevation of sun 40°, crystals vibrating in solar vertical.

of the halo and the zenith distances of the sun and point of contact, respectively,

$$\cos (90° - E') = \cos (90° - E) \cos 21° 50'.$$

Fig. 160 represents, approximately, the outline of the bright band produced in this manner when the elevation of the sun is 40°; making that of the points of contact 36° 38′. O is the position of the observer, S the centre of the halo of 22°, PP parhelia of 22°, TT the points of tangency to this circle of the arcs of Lowitz, PT PT. In order that the arc may reach the halo, the tilt of the crystal must at least equal the elevation of the sun, and

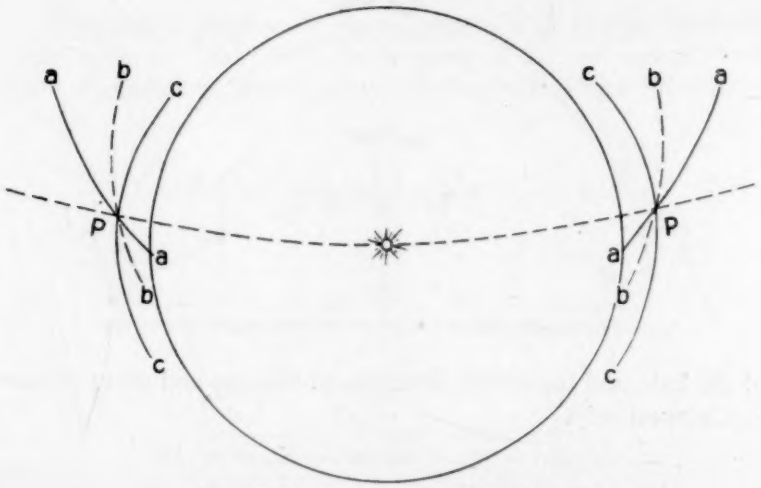
no portion of the lower branch (part below point of tangency) is given unless the tilt is greater than this elevation. Hence, if the extent of the tilting of snow crystals is less, in the great majority of cases, than 30°, as it probably is, only the upper branch is likely to be produced when the sun is 30° or more above the horizon.

Consider, now, the effect of the vibration of the principal axis in a vertical plane at right angles to the vertical plane through the sun. Let E be the elevation of the sun and i the inclination of the principal axis to the vertical, then the angle h between the incident ray and principal plane is given by the equation,

$$\sin h = \sin (90° - i) \sin E.$$

Let $E = 30°$, and $i = 20°$. Then the angular distance from the sun to a parhelion of 22° is 24° 49′, or say 3° from the halo

FIG. 161.



Arcs of Lowitz, crystals vibrating at random.

of 22°. Also $h = 28° 1'$, and the corresponding distance of image from halo is about 2° 40′, at approximately 20°, measured from centre of halo, above or below the parhelion, owing to direction of tip.

Vibrations of the principal axis in intermediate planes give images, of course, in intermediate positions, so that the total

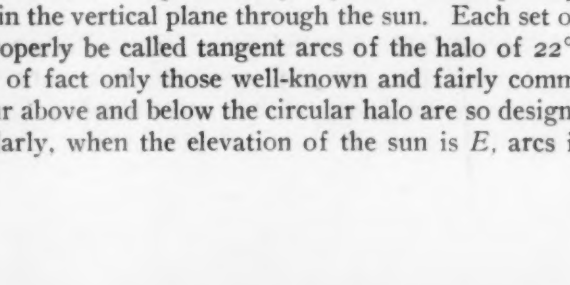
effect, when the elevation of the sun is 30°, may be somewhat as represented in Fig. 161, in which tilting is supposed to be restricted to 30° and less from the vertical.

In this Figure, *PP* are the parhelia of 22°, *aa*, *bb*, and *cc* the outlines of the images corresponding to minimum refraction when the principal axis is oscillated in a vertical plane through the sun, true tangent arcs; at 45°, roughly interpolated, to this sun plane; and at 90° to it, respectively.

It will be noticed that this light, nearly always too faint to be distinguished from the general glare, would be more concentrated, if the orientation of the ice crystals were fortuitous, which, presumably, often is the case, and consequently brighter below the parhelia than above them. Hence, because of limited tilting, as above explained, and because of the greater concentration of light in its lower branches, this halo, whenever seen at all, appears as short arcs including the parhelia and extending mainly below them.

Tangent Arcs of the Halo of 22°.—Obviously, when the sun is on the horizon, ice crystals whose principal axes lie or oscillate

FIG. 162.



Tangent arcs of the halo of 22°.

in horizontal planes must produce the same optical effects above and below (theoretically) the halo of 22° that are produced on its sides, as above explained, by crystals whose principal axes oscillate in the vertical plane through the sun. Each set of curves might properly be called tangent arcs of the halo of 22°, but as a matter of fact only those well-known and fairly common arcs that occur above and below the circular halo are so designated.

Similarly, when the elevation of the sun is *E*, arcs identical

with those just described and tangent to the halo at its highest and lowest points, as shown in Fig. 162, are formed by crystals whose principal axes oscillate in planes normal to the solar vertical and inclined at the angle E to the plane of the horizon.

But ice spicules or needles tend to float with their principal axes horizontal. Hence, it is necessary carefully to determine the optical effects of crystals in this particular position, as may be done by noting the transformations of the tangent arcs as the crystals are so turned as to carry their principal axes from the inclined to the horizontal plane.

Let, then, the principal axis of an ice needle lie parallel to OP (Fig. 162) in which O is the position of the observer, HSH' the inclined plane and S the sun at elevation E. Let h be the inclination of the principal axis to the incident radiation and let a or b be the position of the resulting image. Now, let the crystal, as suggested above, be so turned as to carry its axis from an inclined to a horizontal position, and in such manner as to keep *constant* the angle between the principal axis and the direction of the incident ray. That is, change the direction of the axis from parallel to OP to parallel to OP', with $SP = SP'$. Under these conditions the refracted ray will turn precisely as does the principal axis. Hence, if a' and b' are the new positions of a and b, the angle $aSa' = bSb' = PSP'$.

But from the right spherical triangle OSP'

$$\cos OSP' = \sin PSP' = \tan E \cot SP' = \tan E \tan h,$$

and

$$aSa' = bSb' \text{ arc sin } \tan E \tan h.$$

Since the points of tangency of the " tangent arcs " under consideration are 90° from the corresponding points of the similar " arcs of Lowitz," it follows that the angle B of Fig. 158 and table on page 624 equals TSa (Fig. 162).

Hence,

$$\left.\begin{matrix} TSa' \\ T'Sb' \end{matrix}\right\} = S = B \pm \text{ arc sin } \tan E \tan h$$

The following table, adapted from Pernter-Exner, "Meteorologische Optik," pp. 338–339, gives the necessary data for accurately constructing tangent arcs corresponding to different solar elevations:

Values of Angle S.

E		5°		10°55′		15°		20°		25°2′	
h	Δ	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
0°	21°50′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′
5	21 55	1 21	0 29	1 50	0 0	2 16	−0 26	2 45	−0 55	3 16	− 1 26
10	22 10	2 52	1 06	3 56	0 02	4 42	−0 44	5 40	−1 42	6 42	− 2 44
15	22 38	4 27	1 45	6 04	0 08	7 13	−1 01	8 42	−2 30	10 17	− 4 05
20	23 04	6 15	2 35	8 27	0 25	10 01	−1 11	12 02	−3 12	14 12	− 5 22
25	23 51	8 01	3 21	10 51	0 31	12 52	−1 30	15 27	−4 05	18 16	− 6 54
30	24 49	10 16	4 28	13 46	0 58	16 16	−1 32	19 30	−4 46	23 01	− 8 17
35	26 03	12 53	5 51	17 08	1 36	20 11	−1 27	24 08	−5 24	28 28	− 9 44
40	27 38	16 08	7 42	21 14	2 36	24 55	−1 05	29 42	−5 52	34 59	−11 19
45	29 42	20 25	10 23	26 31	4 17	30 57	−0 09	36 45	−5 57	43 16	−12 28
50	32 26	26 16	14 18	33 34	7 00	38 55	1 39	46 10	−5 26	54 07	−13 33
55	36 26	35 13	20 51	44 02	12 02	50 32	5 32	59 21	−3 17	69 53	−13 49
60	44 38	54 02	36 36	64 50	25 48	72 58	17 40	84 25	6 13	99 19	− 8 41
60°45′	50 04	65 30	47 32	76 40	36 22	85 06	27 56	97 04	15 28	113 02	0 0

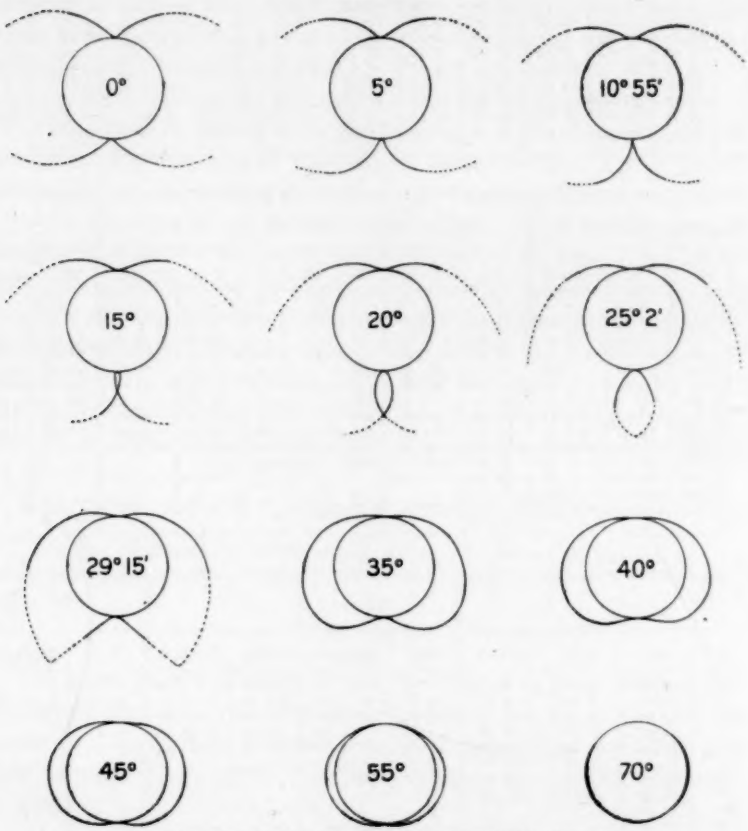
E		29°15′		35°		40°		45°		50°	
h	Δ	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
0°	21°50′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′
5	21 55	3 44	− 1 54	4 26	− 2 36	5 08	− 3 18	5 56	− 4 06	6 54	− 5 04
10	22 10	7 39	− 3 41	9 04	− 5 06	10 30	− 6 32	12 09	− 8 11	14 07	−10 09
15	22 38	11 54	− 5 32	13 51	− 7 43	16 06	− 9 54	18 39	−12 27	21 44	−15 32
20	23 04	16 11	− 7 21	19 11	−10 21	22 12	−13 22	25 46	−16 56	29 08	−21 18
25	23 51	20 49	− 9 27	24 45	−13 23	28 43	−17 21	33 29	−22 07	39 26	−28 04
30	24 49	26 14	−11 30	31 12	−16 28	36 20	−21 36	42 38	−27 54	50 50	−36 06
35	26 03	32 27	−13 43	38 43	−19 59	45 21	−26 37	53 48	−35 04	65 55	−47 11
40	27 38	39 57	−16 07	47 54	−24 04	56 40	−32 50	68 57	−45 07	101 55	−78 05
45	29 42	49 27	−18 39	59 50	−29 02	72 26	−41 38	105 24	−74 36	. . . .	. . . .
50	32 26	62 09	−21 35	76 50	−36 16	110 17	−69 43	. . . .	. . . .	. . . .	. . . .
55	36 26	81 09	−25 07	118 02	−61 58	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
60	44 38	121 15	−30 37	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
60°45′	50 04	146 31	−33 29	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .

E		55°		60°		70°		80°	
h	Δ	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
0°	21°50′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0 ° 0′	0° 0′
5	21 55	8 06	− 6 16	9 38	− 7 48	14 50	−13 00	30 40	−28 50
10	22 10	16 34	−12 36	19 46	−15 48	30 58	−27 00	91 59	−88 01
15	22 38	25 36	−19 28	30 45	−24 33	50 31	−44 19	. . . .	. . . .
20	23 04	35 45	−26 57	41 31	−34 41	94 25	−85 35	. . . .	. . . .
25	23 51	47 26	−36 01	59 34	−48 12	. . . .	. . . .	. . . .	. . . .
30	24 49	62 54	−48 07	97 22	−82 38	. . . .	. . . .	. . . .	. . . .
35	26 03	99 22	−80 38	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
40	27 38	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
45	29 42	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
50	32 26	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
55	36 26	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
60	44 38	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
60°45′	50 04	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .

The upper and lower tangent arcs change with elevation, E, of the sun, as indicated by the table and shown in Fig. 163, copied from Pernter-Exner, "Meteorologische Optik." Portions below the natural horizon can only be seen from sufficient heights.

With increase of elevation of the sun from the horizon the branches of the lower tangent arc draw closer together; become

FIG. 163.



Upper and lower tangent arcs at solar elevations indicated.

pointed; cross, forming a loop; then open out and turn up where they merge with the drooped branches of the upper tangent arc and thus form an enclosing curve, with red inner border, which, at first, when the elevation is 30° to 35°, is bagged below; then, when the elevation is 50° to 60°, approximately elliptical; and,

finally, at elevations of 75° and more, indistinguishably merged with the circular halo. The brightest portions of these arcs are at and near the points of tangency. Hence, when the solar elevation is 35° to 45° the lower arc, if visible over only its brightest part, appears as an inferior tangent arc, which, indeed, it is, circular, perhaps, and limited to, say, 10° to 20°, which, in reality, it is not.

Frequency of Horizontal and Rarity of Vertical Tangent Arcs.—Since the theory of the formation of the horizontal (upper and lower) tangent arcs of the halo of 22° is identical with that of the vertical (arcs of Lowitz), it will be interesting to consider why, though the former is fairly common, the latter is extremely rare. Obviously this must somehow be connected with the attitudes, which radically do differ, of the principal axes of the ice crystals that produce them. The horizontal tangent arcs are produced, as already explained, by crystals whose principal axes are horizontal, and, therefore, by columnar crystals, since these and these alone tend to assume this as an attitude of maximum frequency and thus produce a corresponding concentration of light.

The vertical tangent arcs (arcs of Lowitz), on the other hand, being produced by crystals whose principal axes oscillate in that particular vertical plane that passes through the sun nearly always are too faint to be seen, because, in part, this unique attitude can only rarely be assumed by any considerable proportion of the crystals present. Even the combined effect of the crystals in all vertical planes is seldom noticed because, as explained, of its width and consequent faint nebulosity.

Another factor that probably contributes to the contrast between the frequencies of occurrence of the two types of tangent arcs is the fact that columnar crystals whose principal axes tend to lie horizontal probably are far more effective as refractors, or halo producers, than are the tabular crystals whose principal axes tend to stand vertical. This is because the tabular crystals are so filled with air spaces or other heterogeneities, as shown by their photomicrographs, that anything like regular transmission of light through them from edge to edge is hardly possible. The columns, on the other hand, appear to be more nearly homogeneous and, consequently, much more effective as refractors. This is partially confirmed by the fact that halos often are

seen close at hand in polar regions when the air is filled with ice needles, and rarely if ever seen in ordinary snowstorms consisting essentially of tabular crystals.

The greater efficiency (presumably), then, of the columnar crystal, whose principal axis tends to lie horizontally, as a refractor, over that of the tabular crystal whose principal axis tends to stand vertical, together with the further fact that the orientation of the vertical plane of oscillation must nearly always be fortuitous, seems to explain why the horizontal tangent arc of the halo of 22° is so frequently seen and the vertical so rarely.

It should be remembered, however, that, in apparent contradiction of the above statements concerning the maximum frequency attitudes of the principal axes of ice crystals, there are two special types of columnar crystals whose principal axes tend to stand vertical; namely, those that are shorter than broad (tabular when much shorter than broad), and those that have tabular caps. Perhaps, therefore, all halo phenomena are due essentially to columnar and very little to tabular crystals.

Parhelia of 46°—Since the flat ends of columnar snow crystals and the flat sides of tabular ones both are at right angles to the sides, it follows that optical phenomena must occur that are produced by refraction at such angles, analogous to those already explained for the 60° angle.

Let, then, the 90° intersection be vertical, as it is, more or less, in the case especially of columnar crystals, and let the orientation be that of minimum refraction. If, now, the sun is on the horizon, the distance from it to either of its refraction images, also on the horizon, corresponding to the angle in question, are given by substitution in the general equation,

$$\sin \frac{D_0 + A}{2} = \mu \sin \frac{A}{2}.$$

On putting $\mu = 1.31$, this becomes

$$\sin \left(\frac{D_0}{2} + 45° \right) = 1.31 \sin 45°,$$

from which $D_0 = 45° 44'$.

Hence, these images are known as the parhelia of 46°.

With increase of elevation of the sun the inclination, h, of the incident ray to the vertical face of the crystal is equally increased, as is also the elevations of the images, as we know from previous

considerations. Hence the positions of the parhelia of 46° corresponding to different elevations of the sun may be found in the same manner as those of 22°. On substituting, then, in the equation,

$$\sin \frac{D' + A}{2} = \mu \frac{\cos k}{\cos h} \sin \frac{A}{2},$$

in which $\sin h = \mu \sin k$, the proper values of μ and A, namely 1.31 and 90°, respectively, and also computing the corresponding values of Δ'_0, one obtains the following table:

h	D'_0	Δ'_0
0°	45° 44′	45° 44′
5°	46° 11′	46° 0′
10°	47° 36′	46° 50′
15°	50° 08′	48° 18′
20°	54° 08′	50° 38′
25°	60° 48′	54° 36′
30°	72° 48′	61° 52′
32° 12′	90° 0′	73° 30′

These parhelia, like those of 22°, also trail off parallel to the horizon, for crystals whose attitudes differ somewhat from that of minimum refraction. Such trails, however, necessarily are very faint, and perhaps never observed. In fact, these parhelia themselves are only rarely seen.

Halo of 46°.—Since, as just explained, the image, S_1 (Fig. 164), of the sun produced in the principal plane of a 90° refracting angle of an ice crystal, as seen by the observer, O_1, is 45° 44′ from the sun, S, itself ($\mu = 1.31$), it follows that when such crystals are very abundant and set at random in all directions the innumerable images so produced must together assume the shape of a ring about the sun of radius 45° 44′. This is the well-known, though not very common, halo of 46°.

Whenever at all conspicuous, this halo also shows colors (red being nearest the sun) which, because of the greater dispersion produced by the angle of 90° than by the angle of 60°, are more widely separated than in the halo of 22°. Hence, it likewise has the greater width of the two—about 2° 36′, corresponding to the diameter of the sun, 30′, plus the dispersion, that is, to $30' + D_v$, $(\mu = 1.317) - D_r$, $(\mu = 1.307) = 30' + 47° 16' - 45° 10' = 2° 36'$.

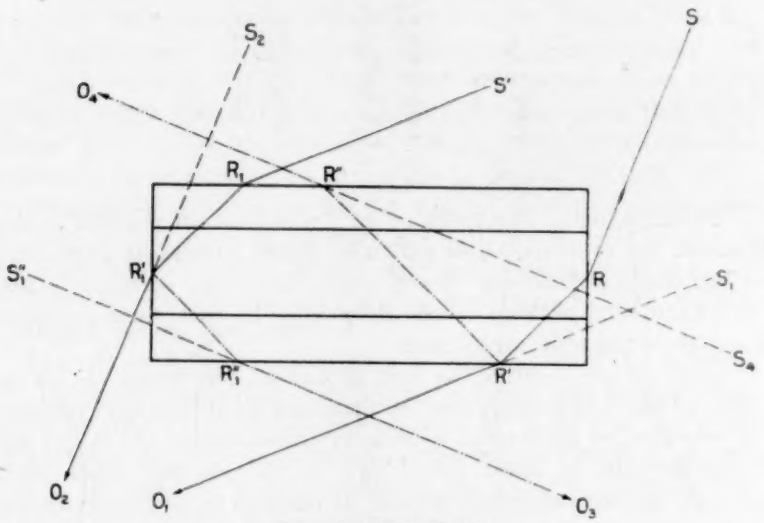
In addition to the colored ring, there is also a broad outer band of diffuse white light corresponding to all refractions other

than the minimum, but it is too faint and too uniformly distributed to be conspicuous, or, perhaps, unmistakably seen even when carefully looked for.

Halo of 90°.—Occasionally, a faint white halo, sometimes called the halo of Hevelius, is seen at 90° from the sun.

Several explanations of this halo have been suggested, but none gives it the right distance from the sun or is otherwise satisfactory. The following simple theory of its formation, therefore, is offered, based on the presence of fortuitously directed columnar crystals—short columns. Let a ray of whatever

FIG. 164.



Formation of halo of 46°.

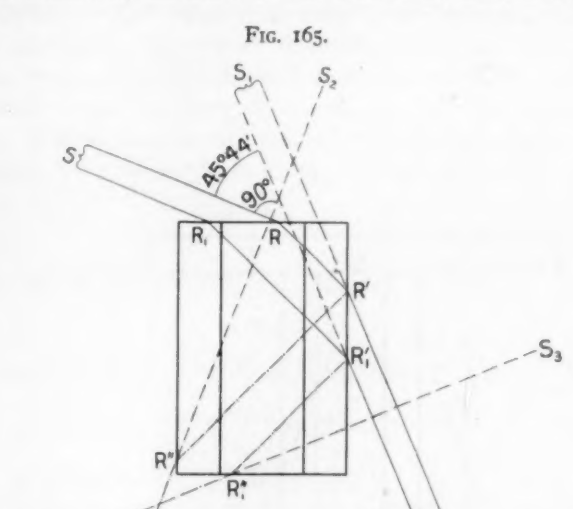
wave-length be incident at R, say (Fig. 165), on the flat end of a columnar crystal in the direction appropriate to *minimum refraction,* and let most of that portion of it reflected at R' emerge from the opposite face at R''. The resulting image, S_2, seen by an observer at O_2, clearly is 90° from S. And since this direction holds for the more concentrated (minimum refracted) portion of every color, it follows that the resulting 90° halo must be white. It also must be more sharply defined on the side away from than on the side next to the sun.

Bouguer's Halo, White Halo of 136°.—Rays entering a hori-

zontal crystal face at R_1 and emerging at R''_1 (Figs. 164 and 165) clearly must produce a faint white image at about 135° 44′ ($\mu = 1.31$) from the sun, and a cloud of such horizontal crystals a white halo of outer radius 44° 16′ and inner radius much less about the antisolar point as a centre. Perhaps this is the Bouguer's halo, or false white rainbow, the theory of which has been considered obscure.

Circumzenithal Arc.—Occasionally, an arc of, perhaps, 90°, having its centre at the zenith, and, therefore, known as the cir-

FIG. 165.



Formation of the white halo of 90°.

cumzenithal arc, is seen some 46°, or a little more, above the sun. It generally lasts only a few minutes, about five on the average, but during that time often is so brilliantly colored, especially along that portion nearest the sun—red on the outside, to violet inclusive—as to be mistaken by persons unfamiliar with it for an exceptionally bright rainbow. It occurs most frequently when the altitude of the sun is about 20° and at times when the parhelia of 22° are conspicuous. Presumably, therefore, when the principal axes of a large portion of the crystals are practically vertical. That is, when the snow crystals are largely columnar

with tabular caps, a moderately common form,[193] as outlined in Fig. 166.

The explanation of this halo, as of many others, was first given by Bravais,[194] and is very simple.

In still or steadily flowing air the type of crystals assumed

FIG. 166.

Formation of circumzenithal arc.

will keep their principal axes substantially vertical. Let, then, H (Fig. 166) be the altitude of the sun above the horizon, and also the angle between the incident ray and the upper horizontal surface of the crystal; let θ be the angle between this surface and

[193] Bentley, *Monthly Weather Review*, **30**, p. 609, 1902.

[194] Mémoire sur les halos, *Journal de l'École polytechnique*, 31me cahier, 1845.

the refracted ray, and let H' be the altitude of the solar image produced by the 90° prism.

Hence,

$$\cos H = \mu \cos \theta,$$

and

$$\sin H' = \mu \sin \theta = \sqrt{\mu^2 - \cos^2 H}.$$

As previously explained, the angles between a principal plane (plane normal to the refracting edge) and incident and emergent rays are equal. Therefore, since the position of the image is altered by rotation of the crystal about its principal axis while its altitude remains unchanged, it follows that the halo so produced is a limited circular arc whose centre is the zenith.

From the equation for minimum refraction,

$$\sin \frac{D_0 + A}{2} = \mu \sin \frac{A}{2},$$

it appears that in the present case, and for $\mu = 1.31$,

$$D_0 = 45° 44'.$$

This corresponds to $H = 22° 8'$, and to $H' = 67° 52'$. In this case

$$H' - H = 45° 44',$$

which is the radius of the 46° halo.

With increase or decrease of the altitude of the sun from 22° 8′, the solar distance of the circumzenithal arc increases, but so slowly at first that the gain amounts to only about 1° when the sun has sunk to 16°, or risen to 27°. Hence, this arc is also, though erroneously, called the upper tangent arc of the halo of 46°.

When the sun is on the horizon the solar distance of the circumzenithal arc is 57° 48′, and the interval between it and the 46° halo 12° 4′. On the other hand, the arc rapidly converges on the zenith as the altitude of the sun approaches 32°, and is theoretically impossible for solar altitudes greater than 32° 12′.

Kern's Arc.—Kern's arc, so designated from the name of the first observer to report [195] it, occurs exactly opposite the corre-

[195] *Koninklijk nederlandsch meteorologisch Instituut. Onweders, optische verschijn., enz., in Nederland,* 1895, O. 66.

sponding circumzenithal arc and on the same circle. It might, therefore, also be called the anticircumzenithal arc.

No explanation of this arc is at hand. Its theory, however, appears to be identical with that of the circumzenithal, with the single exception that while the latter is produced by light transmitted at R (Fig. 166) the former, which is much the fainter of the two,[196] is due to that portion of the light which is there reflected, thus producing the image S''.

Circumhorizontal Arc.—A colored arc, red on the upper side, of perhaps 90° in extent, is occasionally seen parallel to the horizon and about 46°, or a little more, below the sun. This arc is produced by light entering snow crystals through vertical sides and passing out through horizontal bases, and, therefore, the theory of its formation is identical with that of the circumzenithal arc. On merely substituting "zenith distance" for "elevation" all the numerical values of the one become those of the other. Hence, the circumzenithal arc cannot appear when the zenith distance of the sun is greater than 32° 12′. Similarly, when this distance is 22° 8′, corresponding to minimum refraction, the solar distance of the circumhorizontal arc is 45° 44′, the radius of the halo of 46°. Further, for all values of the zenith distance of the sun from 16° to 27° the circumhorizontal arc is within 1° of tangency to the halo of 46°. Hence, it is also, though incorrectly, called the lower tangent arc of the halo of 46°.

Lateral Tangent Arcs of the Halo of 46°.—Just as flat-topped crystals with vertical sides produce a circumzenithal arc when the altitude of the sun is between 0° and 32° 12′, so, too, similar crystals whose axes are horizontal and directed towards any point whose solar distance is between 90° and 57° 48′, or between 0° and 32° 12′, produce a colored arc—red next the sun—about this directive point as a centre. And as there are two such points corresponding to each solar distance, one to the right, the other to the left, of the solar vertical, it follows that arcs formed in the above manner are symmetrically situated with respect to this vertical. Further, when the solar distance of the directive point is 67° 52′ or 22° 8′, the resulting arc is tangent to the halo of 46°, and as always some, at least, of the innumerable crystals are turned towards this point, except when the altitude of the sun is greater than these values, respectively, it follows, with the

[196] *Monthly Weather Review*, **34**, p. 124, 1906.

same exceptions, that the blend of the numerous arcs produced by the various directed crystals is always tangent to the halo of 46°, and also that except near the point of tangency only the red of these blends is reasonably pure.

Obviously, there are two classes of lateral tangent arcs, namely, lower, as seen at S_1 by an observer at O_1 (Fig. 164), and upper, as seen at S_2 by an observer at O_2. These will next be considered separately as infralateral and supralateral arcs.

Infralateral Tangent Arcs of the Halo of 46°.—Let the circle about S (Fig. 167) be the halo of 46°; let the altitude, H, of the sun be less than 67° 52′, and let the principal axes of the columnar crystals be horizontal and directed towards the point, P, on the horizon distant 67° 52′ from S. As previously explained, the infralateral tangent arcs will be tangent to this halo at the point T, where it is intersected by the arc SP. The position of T may easily be determined from the value of the angle A, between the vertical SB and the arc SP.

Obviously, from the right spherical triangle, SBP,

$$\cos A = \tan H \cot 67° 52'.$$

Since refraction by the crystal is limited to solar distances of P between 57° 48′ and 90°, it follows that A_1 and A_2, corresponding to the lower and upper ends of the tangent arc, are given by the equations,

$$\cos A_1 = \tan H \cot 57° 48'$$

and

$$A_2 = 90°,$$

respectively.

When the altitude of the sun is 57° 48′, or a little greater, the two tangent arcs, springing from a common point on the solar vertical, form a wide V.

When the solar altitude equals 67° 52′, the two arcs, now merged into a smooth continuous curve, are tangent to the halo at its lowest point.

Finally, for altitudes of the sun greater than 67° 52′, the arcs, still appearing as a single curve, are slightly separated from the circular halo even at its lowest and closest point.

Supralateral Tangent Arcs.—When the altitude of the sun is less than 22° 8′ supralateral tangent arcs similar to the infralateral are produced.

The point of tangency of the supralateral tangent arc with the halo of 46° is given in terms of the arc, A, on this halo from its upper point.

When the solar altitude, H, is less than 22° 8′,

$$\cos A = \tan H \cot 22° 8'.$$

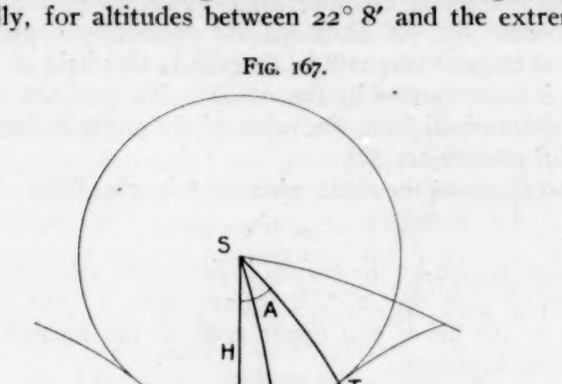
Similarly, the possible end of the arc is given by the equation,

$$\cos A' = \tan H \cot 32° 12'.$$

When the altitude of the sun is 22° 8′, both arcs, forming a continuous curve, are tangent to the halo at its highest point.

Finally, for altitudes between 22° 8′ and the extreme limit,

FIG. 167.



Formation of the lateral tangent arcs of the halo of 46°.

32° 12′, these arcs are more or less above the halo of 46°.

The following table gives the value of A for different altitudes of the sun.

Infralateral arcs.		Supralateral arcs.	
H	A	H	A
0°	90° 0′	0°	90° 0′
10°	85° 54′	5°	77° 35′
20°	81° 35′	10°	64° 18′
30°	76° 25′	15°	48° 47′
40°	70° 03′	20°	26° 30′
50°	61° 00′	22° 8′	0° 0′
60°	45° 13′	25°	0° 0′
67° 52′	0° 0′	30°	0° 0′
		32° 12′	0° 0′

Halos of Unusual Radii.—Halos of various radii other than those already given have occasionally been reported. They can readily be accounted for on the assumption that the columnar ice crystals have certain pyramidal bases that afford the appropriate refraction angles.

Secondary Halos.—Obviously, each bright spot of the primary halo phenomena, especially the upper and lower points of the 22° circle and its parhelia, must in turn be the source of secondary halos. Doubtless, the 22° halos of the lateral parhelia contribute much to the flaring vertical column through the sun that occasionally has been seen; and, perhaps, the brilliant upper and lower points of the halo of 22° may produce faint secondary parhelic circles. In general, however, very few of the secondary halos are ever bright enough to be seen even when carefully looked for.

Singular Halos.—A few halos not included in any of the above classes have been *once* reported. No satisfactory explanations of them have been offered. Clearly, though, since the ice crystal appears in many modified forms—with flat tubular, and pyramidal ends, for instance—and even in orderly clusters, it is obvious that although only a few halos are well known, a great many are possible.

CHAPTER V.

REFLECTION PHENOMENA.

A FEW optical phenomena of the atmosphere, usually classed as halos, are produced by simple reflection.

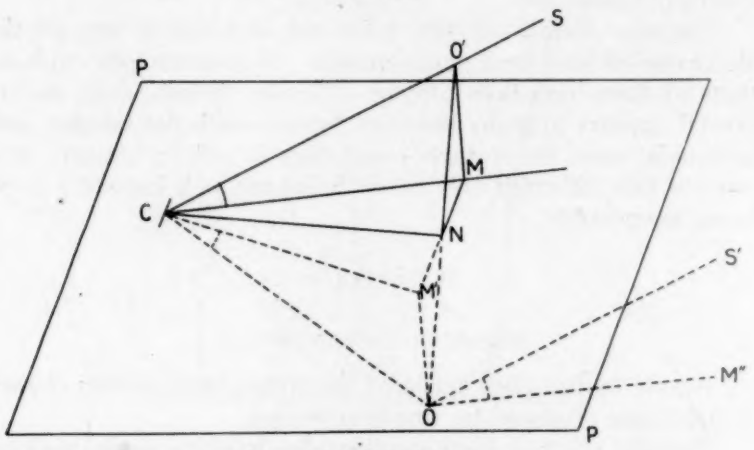
Parhelic Circle.—Very occasionally, a white circle, perhaps faint and tending to be diffuse, passes through the sun parallel to the horizon and, therefore, crosses the positions of the parhelia, anthelion, paranthelia, etc. This circle is produced by simple reflection (hence it is white) from vertical faces of ice crystals, as may easily be demonstrated.

To this end, let PP (Fig. 168) be a plane parallel to the horizon and normal to a vertical face of an ice crystal at C. Let SC be an incident ray reflected to the observer at O. Let CN be normal to the reflecting face, and lie in the plane PP. By the laws of reflection, CS, CN, and CO lie in a common plane, and the angle SCN = the angle OCN. If, now, $CO' = CO$,

and $O'M$ and OM' be drawn normal to the plane PP, it is obvious that the triangle $NMO' =$ the triangle $NM'O$, and that $MO' = M'O$. Hence, as the angles CMO' and $CM'O$ are right angles, and $CO' = CO$, the angle MCO', that is, the elevation of the sun above the plane PP, or above the plane of the horizon to which PP is parallel = the angle $M'CO$, the angle of depression at C of O below the plane PP, or angle of elevation of the crystal, C, above the observer's horizon.

But C is the position of any *vertical* face that reflects light to the observer at O. Hence, the parhelic circle passes through the sun and is parallel to the horizon. Hence, also, it is superimposed upon such parhelia and parhelic tails as may coexist. Occasionally, therefore, one may be unable to distinguish between a

FIG. 168.



Formation of the parhelic circle.

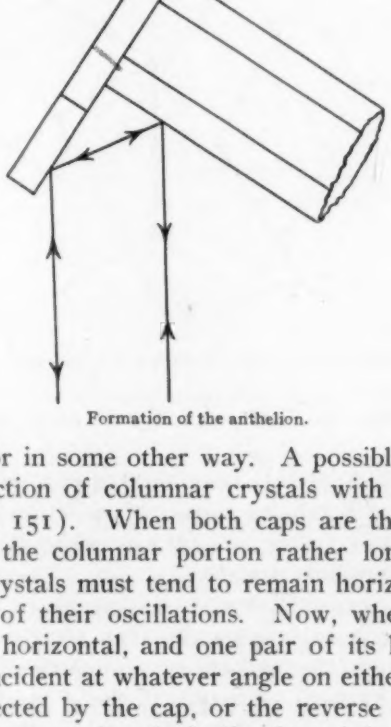
portion of a parhelic tail and a segment of the parhelic circle. There can be no doubt, however, in regard to any portion that occurs nearer the sun than the appropriate positions of the parhelia of 22°. Such portion cannot be produced by refraction, and, consequently, must belong to the parhelic circle.

Since the formation of this circle requires a predominance of vertical faces, and since a simple columnar hexagonal prism tends not only to keep its major axis horizontal but also one of its faces down, it follows that the required vertical surfaces must be either the flat ends of such crystals, or, most probably, the hex-

agonal faces of columnar crystals with tabular capped ends (Fig. 151). In the latter case, parhelia are also very likely to appear, while in the former they are apt to be entirely absent, and even the parhelic circle itself too faint to be clearly observed.

Anthelion.—On rare occasions a bright white spot, known as the anthelion, is seen on the parhelic circle opposite the sun. Of course, all crystals that contribute to the production of the parhelic circle also add to the brightness of the anthelion, but as the latter sometimes occurs when the circle is inconspicuous, or not even seen, it follows that the supplementary light must

FIG. 169.



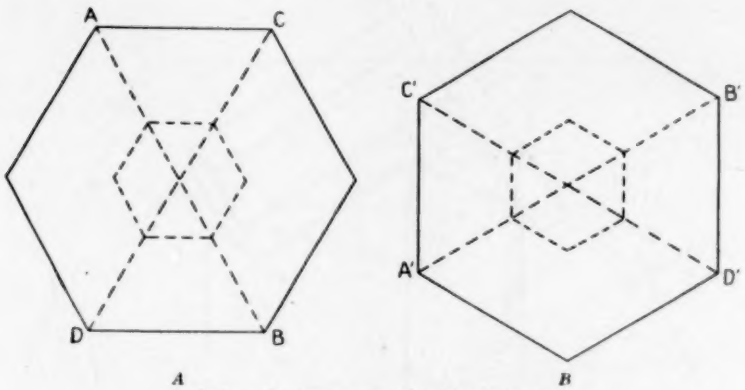
Formation of the anthelion.

be accounted for in some other way. A possible explanation is found in the action of columnar crystals with tabular caps at each end (Fig. 151). When both caps are the same size, or nearly so, and the columnar portion rather long the principal axes of such crystals must tend to remain horizontal, whatever the amplitudes of their oscillations. Now, whenever a crystal of this kind is horizontal, and one pair of its hexagonal faces vertical, light incident at whatever angle on either of these faces and finally reflected by the cap, or the reverse (see Fig. 169), will take such a course that the inclination of the reflected ray to

the plane of the horizon will be the same as that of the incident ray, and the projected directions on this plane exactly opposite to each other. Hence, this type of crystal will send much light to an observer from that point of the parhelic circle directly opposite the sun, enough, perhaps, when such crystals are particularly abundant, to produce a noticeably bright anthelion.

Oblique Arcs of the Anthelion.—On several occasions from one to four oblique white arcs have been seen to pass through the anthelion. Usually, when they occur, there are two such arcs symmetrically placed on either side of the vertical. The cause of these arcs is not definitely known, but as they are white (only once reported to show colors) it appears that presumably they are owing to reflection.

FIG. 170.



End views of columnar crystals with tabular caps.

Possibly some, at least, of these arcs may be caused as follows:

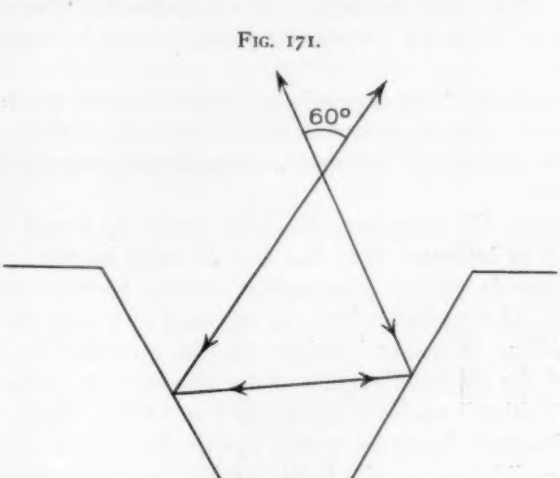
Let Fig. 170 A and B be end-on views of horizontal columnar crystals with tabular caps. A's position seems to be the stablest for crystals falling in still air, while B's position, perhaps, is one of secondary stability.

If, now, these crystals should oscillate about their diagonals, *AB, CD,* etc., as possibly they do, they clearly would produce white arcs through the anthelion, symmetrically arranged on either side of its vertical. Crystals in the *A* position would give arcs diverging 120° from each other, while the angle between

the arcs produced by crystals in the *B* position would be only half as great, or 60°.

Parhelia of 120°.—Bright spots that show no trace of color are occasionally seen on the parhelic circle 120° from the sun, in azimuth, or 60° from the anthelion. These are known as the parhelia of 120°, and while their origin is not certain, the fact that they are white is well nigh conclusive evidence that they are somehow produced by simple reflection, possibly from the faces of reëntrant angles.

FIG. 171.



Formation of parhelia of 60°.

These angles, which are of frequent occurrence in tabular crystals, are of two values, 60° and 120°, respectively, as shown in Figs. 171 and 172. And since such crystals fall more or less flatwise, the polygonal faces tend to remain vertical. Hence, when the crystals are rather thick and the faces of the reëntrant angles, therefore, sufficiently broad, light incident on one such face may be reflected to its companion and then from it in turn in such direction that the projections of the initial and final rays onto the plane of the horizon will differ in direction by 120°, as is obvious from Figs. 171 and 172. Both angles produce the

same effect, though, owing to the value of the angles of incidence involved, the 120° one presumably is the most effective.

Parhelia of 90°.—The brightest spots that on rare occasions are on the parhelic circle midway between the sun and the anthelion, that is, the parhelia of 90° probably are owing merely to the intersection of this circle with the halo of 90°. Both circles are white, and, therefore, the brighter spots produced by their intersection are also white.

Pillars.—During very cold weather vertical pillars of white light are often seen extending above and below the sun, when its elevation is small, or merely rising above it, when it is on the horizon.

The upper and lower portions of these pillars, counting from the sun, are owing, as has long been known, merely to reflection by the under and the upper surfaces, respectively, of tabular ice crystals.

Crosses.—On very rare occasions strips of white light have been seen to intersect over the sun at right angles. This rare phenomenon is owing, presumably, merely to the simultaneous occurrence of a parhelic circle, or segment of it next the sun, and a light pillar. Possibly it might also be produced by the intersection of the secondary halos of 22°, or even by some combination of "pillar," parhelic circle, and secondary halos. A competent observer, however, could easily distinguish between the several possible causes of a light "cross," and thus determine the actual origin of any particular instance of this phenomenon he might happen to see.

CHAPTER VI.

DIFFRACTION PHENOMENA.

Coronas.—Coronas consist of one or more sets of rainbow-colored rings, usually of only a few degrees radius, concentrically surrounding the sun, moon, or other bright object when covered by a thin cloud veil. They differ from halos in having smaller (except in rare cases) and variable radii, and in having the reverse order of colors; that is, blue nearest the sun, say, and red farthest away.

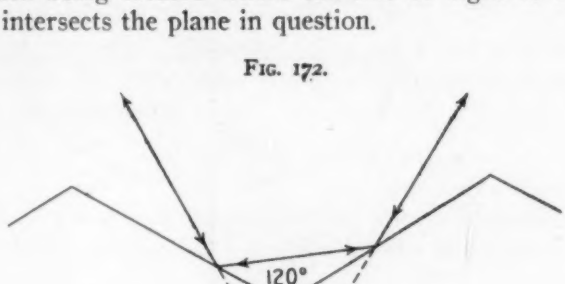
Clearly, then, coronas are caused by diffraction, or the distribution of effective (non-neutralizing) quantities of light off

the primary path, resulting from the action of cloud particles on radiation incident from a distant source.

Consider, then, the diffractive action of a layer of innumerable water droplets on a parallel beam of monochromatic light.

In this case the wave front, or continuous locus of any given phase, is flat—pits and pimples on it would quickly be smoothed out by dispersion—and everywhere normal to the line of travel. Also the droplets, because of their very short focal lengths and consequent great dispersive power, affect the parallel beam substantially as would so many opaque disks each of the size of a great circle of the corresponding droplet and normal to the line of travel. Furthermore, as the incident light is parallel the centres of the droplets may be regarded as lying in a common plane, each being located where the line of sight to its actual position intersects the plane in question.

FIG. 172.



Formation of parhelia of 120°.

The problem, then, reduces to that of finding the diffraction pattern produced in an isotropic transparent medium by a great many irregularly distributed opaque disks on a plane wave front of monochromatic light.

For the solution of this problem, it is convenient to make use of the fact, first explained by Huygens, and later developed by Fresnel, Stokes, Rayleigh and others (see any good work on optics), that each point in a wave front may itself be regarded as a secondary source of light of the same color.

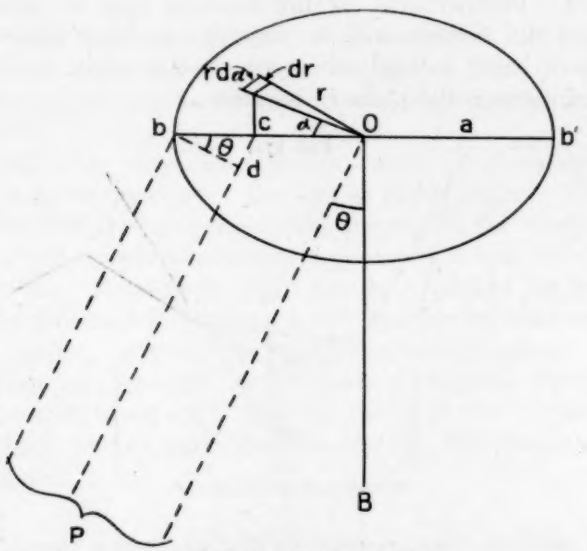
According to Stokes,[197] the intensity in different directions of the secondary radiation is proportional to $1 + \cos\theta$, in which

[197] *Math. and Phys. Papers*, vol. ii, p. 243; *Camb. Phil. Trans.*, 9, 1, 1849.

θ is the angle of deviation from the course of the parent light. That is, it is symmetrically distributed about the normal to the wave front at the secondary source, with its maximum value straight forward and minimum (zero) directly back.

A further aid to the solution of this problem is furnished by Babinet's principle, which may be explained as follows: When parallel light passes through a large opening in an opaque screen, but little illumination occurs outside the primary beam, and that little owing to rays from the edge of the opening whose angle of deviation from the original direction is very small. If,

FIG. 173.



Diffraction by a circular opening.

now, this large opening should be partially covered by a great many opaque disks, as assumed in the problem under consideration, and if illumination should result, as it does, at places where formerly there was complete shadow, then, if the opaque disks should become transparent and the transparent interspaces opaque, precisely the same illumination in the "shadow region" would obtain as before, but in exactly the opposite phase, as is obvious from the fact that the two illuminations acting together produce darkness.

Babinet's principle, therefore, enables one to use circular

openings and opaque disks interchangeably in the solution of diffraction problems. And as it is easier to discuss the circle mathematically than an irregular area the above problem will be substituted by its physical equivalent. That is, circular openings in an opaque screen will be substituted for opaque disks on a wave front.

Let O (Fig. 173) be the centre of a small circular opening of radius a in an opaque screen, and let parallel light pass through this opening, normal to its plane, in the direction OB. Let the plane fixed by P, the point whose illumination is to be determined, and the line OB intersect the circle in the diameter bb', and let the angle $BOP = \theta$. Finally, let r be the distance of any element of the circle from the centre, O, c, the foot of the perpendicular from this element onto the diameter bb', and α the angle between r and the radius Ob.

Clearly, then, the difference of phase at P between light from an element of the wave front at b and from any other element is given by the expression

$$2\pi \frac{cd}{\lambda}, \text{ or } \frac{2\pi}{\lambda} (a - r \cos) a \sin \theta,$$

in which λ is the wave-length. Also, since the displacement at P owing to the element at b is given by the expression

$$\sin 2\pi \frac{t}{T} r \, da \, dr.$$

in which t is the time of travel from b to P and T the period, the total displacement, X, at P is given by the equation,

$$X = \int_0^{2\pi} \int_0^a \sin 2\pi \left[\left(\frac{t}{T} - \frac{a \sin \theta}{\lambda} \right) + \frac{r \sin \theta}{\lambda} \cos \alpha \right] r \, da \, dr.$$

Hence, developing, and putting

$$\int_0^{2\pi} \int_0^a \cos 2\pi \left(\frac{r}{\lambda} \sin \theta \cos \alpha \right) r \, da \, dr = A$$

and

$$\int_0^{2\pi} \int_0^a \sin 2\pi \left(\frac{r}{\lambda} \sin \theta \cos \alpha \right) r \, da \, dr = B$$

$$X = A \sin 2\pi \left(\frac{T}{t} - \frac{a \sin \theta}{\lambda} \right) + B \cos 2\pi \left(\frac{t}{T} - \frac{a \sin \theta}{\lambda} \right).$$

Therefore, A and B are components at right angles to each other of the resultant amplitude. Hence, the intensity, I, is given by the equation,

$$I = A^2 + B^2.$$

Putting

$$\frac{\pi r \sin\theta}{\lambda} = \beta r$$

$$A = \int_0^{2\pi}\int_0^{a} \cos(2\beta r \cos\alpha)\, d\alpha\, r\, dr$$

$$= \int_0^{2\pi}\int_0^{a} \left[1 - \frac{(2\beta r\cos\alpha)^2}{1.2} + \frac{(2\beta r\cos\alpha)^4}{1\cdot2\cdot3\cdot4} - \cdots\right] d\alpha\, r\, dr$$

But

$$\int_0^{2\pi} \cos^{2n}\alpha d\alpha = \frac{1\cdot3\cdot5\cdot\ldots\ldots 2n-1}{2\cdot4\cdot6\cdot\ldots\ldots 2n}\, 2\pi$$

Hence,

$$A = \pi\int_0^{a}\left[2r\,dr - \frac{2\beta^2 r^3\,dr}{1} + \frac{2\beta^4 r^5\,dr}{(1\cdot2)^2} - \frac{2\beta^6 r^7\,dr}{(1\cdot2\cdot3)^2} + \cdots\right]$$

$$= \pi\left[a^2 - \tfrac{1}{2}\frac{\beta^2 a^4}{1} + \frac{1}{3}\frac{\beta^4 a^6}{(1\cdot2)^2} - \frac{1}{4}\frac{\beta^6 a^8}{(1\cdot2\cdot3)^2} + \cdots\right]$$

and, putting

$$\beta a = \frac{\pi a \sin\theta}{\lambda} = m.$$

$$A^2 = \pi^2 a^4\left[1 - \tfrac{1}{2}\frac{m^2}{1} + \frac{1}{3}\frac{m^4}{(1\cdot2)^2} - \frac{1}{4}\frac{m^6}{(1\cdot2\cdot3)^2} + \cdots\right]^2$$

A similar development gives the other component in terms of a series of odd valued sines of α. Hence, as the elements are symmetrically distributed on either side of the diagonal bb', $B = O$, and

$$I = A^2.$$

On giving m various values, tables and curves of intensity may be constructed. The following table by Airy, copied from Mascart's "Traité d'optique," v. I, p. 310, is restricted to that portion of the expression within the brackets.

The following table of diffraction maxima and minima is also copied from Mascart, *l.c.*, p. 312.

m	A	I	m	A	I
0.0	1.0000	1.0000	3.0	−0.0922	0.0085
0.1	0.9950	0.9900	3.1	−0.0751	0.0056
0.2	0.9801	0.9606	3.2	−0.0568	0.0032
0.3	0.9557	0.9134	3.3	−0.0379	0.0014
0.4	0.9221	0.8503	3.4	−0.0192	0.0004
0.5	0.8801	0.7746	3.5	−0.0013	0.0000
0.6	0.8305	0.6897	3.6	0.0151	0.0002
0.7	0.7742	0.5994	3.7	0.0296	0.0009
0.8	0.7124	0.5075	3.8	0.0419	0.0017
0.9	0.6461	0.4174	3.9	0.0516	0.0027
1.0	0.5767	0.3326	4.0	0.0587	0.0035
1.1	0.5054	0.2554	4.1	0.0629	0.0040
1.2	0.4335	0.1879	4.2	0.0645	0.0042
1.3	0.3622	0.1312	4.3	0.0634	0.0040
1.4	0.2927	0.0857	4.4	0.0600	0.0036
1.5	0.2261	0.0511	4.5	0.0545	0.0030
1.6	0.1633	0.0267	4.6	0.0473	0.0022
1.7	0.1054	0.0111	4.7	0.0387	0.0015
1.8	0.0530	0.0028	4.8	0.0291	0.0008
1.9	0.0067	0.0000	4.9	0.0190	0.0004
2.0	−0.0330	0.0011	5.0	0.0087	0.0001
2.1	−0.0660	0.0044	5.1	−0.0013	0.0000
2.2	−0.0922	0.0085	5.2	−0.0107	0.0001
2.3	−0.1116	0.0125	5.3	−0.0191	0.0004
2.4	−0.1244	0.0155	5.4	−0.0263	0.0007
2.5	−0.1310	0.0172	5.5	−0.0321	0.0010
2.6	−0.1320	0.0174	5.6	−0.0364	0.0013
2.7	−0.1279	0.0164	5.7	−0.0390	0.0015
2.8	−0.1194	0.0143	5.8	−0.0400	0.0016
2.9	−0.1073	0.0115	5.9	−0.0394	0.0016
3.0	−0.0922	0.0085	6.0	−0.0372	0.0014

Diffraction Maxima and Minima.

	Maxima			Minima		
	$\frac{m}{\pi}$	Diff.	I	$\frac{m}{\pi}$	Diff.	I
1	0.000		1.00000	0.610		0
		0.819			0.506	
2	0.819		0.01745	1.116		0
		0.527			0.503	
3	1.346		0.00415	1.619		0
		0.512			0.502	
4	1.858		0.00165	2.121		0
		0.504			0.501	
5	2.362		0.00078	2.622		0
		0.500			0.500	
6	2.862		0.00043	2.122		0
		0.500			0.500	
7	3.362		0.00027	3.622		0
		0.500			0.501	
8	3.862		0.00018	4.123		0
		0.500			0.500	
9	4.362		0.00012	5.623		0

It will be noticed that the decrease of intensity from maximum to minimum, though large at first, quickly becomes very small.

From the values of $\frac{m}{\pi}$, corresponding to diffraction minima, it is evident that

$$\sin\theta = (n + 0.22)\frac{\lambda}{2a}, \text{ very nearly,}$$

in which n is the order of the minimum, counting from the centre.

This important equation gives the angular distance from the light source at which the successive diffraction minima occur for any particular wave-length and size of drop or disk. It also gives the diameter of the drop when the wave-length, angular distance from the centre, and order of the minimum are known. Furthermore, it shows that the larger the wave-length and the smaller the droplet the larger the diffraction circle, or halo.

The above discussion applies to a single circular disk on the wave front. An exact duplicate disk obviously would produce an exact duplicate diffraction pattern. If, then, two such disks occur close together, and if the distance between their centres, or other homologous points is b, and ϕ the angle between the line connecting these points and the line connecting the farthest to the point of observation, then the difference in phase between the two lights at the latter place is $2\pi\frac{b\sin\phi}{\lambda}$, and a secondary diffraction pattern, in addition to the two primary circular ones, is produced, directed at right angles to the line connecting the centres of the disks. Similarly, conspicuous diffraction patterns are produced by any regular geometric distribution of many disks.

Let, however, the disks, or droplets, be numerous, irregularly distributed, and all of the same size (if of various sizes their effects cannot easily be summed up). Let each produce at a given point a disturbance whose amplitude is A, but let the phases be $\varepsilon_1, \varepsilon_2 - - - - - \varepsilon_n$, and let R be the resultant amplitude.

Then,

$$R^2 = \Sigma\,(A\cos\varepsilon)^2 + \Sigma\,(A\sin\varepsilon)^2,$$

$$= A^2\left[(\cos\varepsilon_1 + \cos\varepsilon_2 + - - - + \cos\varepsilon_n)^2 + (\sin\varepsilon_1 + \sin\varepsilon_2 + - . - . + \sin\varepsilon_n)^2\right],$$

$$R^2 = A^2\,(n\cos^2\varepsilon + n\sin^2\varepsilon + 2n\cos\varepsilon\cos\varepsilon' + 2n\sin\varepsilon\sin\varepsilon'),$$

$$= A^2 n + 2A^2 n\cos(\varepsilon - \varepsilon').$$

But as n is large and the disks irregularly scattered, it is clear that the phase difference, $\epsilon - \epsilon'$, between the innumerable pairs will have all manner of values with, on the whole, the positive and negative well balanced. Hence, as close as can be detected,

$$R^2 = nA^2.$$

That is, the diffraction rings, corona, for instance, produced by a large number, n, of irregularly distributed neighboring droplets are the same as those produced by any one of them, but n times as bright.

When the incident light is complex, the diffraction pattern produced by the several wave-lengths necessarily overlap and produce correspondingly colored rings—red, if present, being the outermost, and blue the innermost.

Size of Cloud Particles.—Since the diffraction pattern produced by a great many irregularly distributed droplets of uniform size is the same as that due to a single one, it is clear that the size of the cloud particles producing coronas may be determined by the equation,

$$\sin \theta = (n + 0.22) \frac{\lambda}{2a},$$

in which, as already explained, θ is the angular distance from the centre of the corona to the nth minimum corresponding to light of the wave-length, λ, and a, the radius to be determined.

Measurements made in this manner have shown that the radii of corona-producing cloud droplets, though varying over a considerable range, commonly average about .007 mm. to .010 mm.

It may be interesting to note in this connection that a contracting or decreasing corona implies growing droplets and, perhaps, the approach of rain; and that an expanding corona implies, on the other hand, decreasing or evaporating droplets and, presumably, the approach of fair weather.

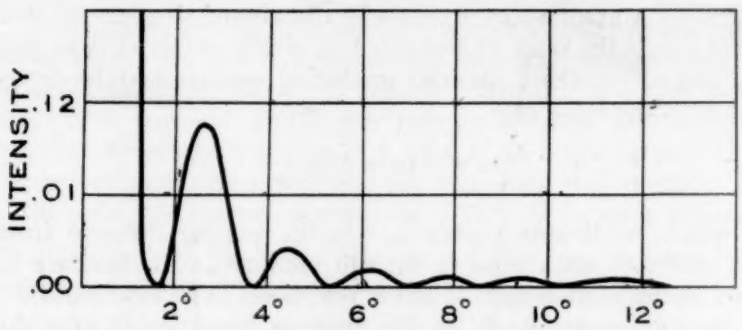
Fig. 174, copied from an instructive article by Simpson,[198] gives the angular and intensity distribution of the monochromatic light $\lambda = .000571$ mm. in a corona produced by droplets of .01 mm. radius.

Droplets versus Ice Needles as Producers of Coronas.—When

[198] Q. *Jr. Roy. Met. Soc.*, **38**, p. 291, 1912.

coronas are seen in clouds whose temperature is above 0° C., or in which halos do not form, it is certain that they are due to droplets. It is well known, however, that the most brilliant coronas—those of multiple rings and large diameter—usually are formed by very high clouds whose temperature often must be far below freezing. Naturally, then, it has been inferred that these coronas are produced by the diffractive action of ice needles. Simpson,[199] however, appears to have disproved the probability that they are formed in this manner. "On no occasion," he says, referring to his stay in the Antarctic, "were a corona and halo seen at the same time on the same cloud." Furthermore, he explains, as the axes of the needles are essentially horizontal, this being their stable position, only those

FIG. 174.



Distribution of intensity by droplets, radius = 0.01 mm., $\lambda = .571\ \mu$.

at right angles to radii from the sun, or other luminary, could produce coronas of the kind observed, while the equally numerous crystals of every other orientation would produce such different patterns that the total effect probably could be but little more than white light—certainly nothing approaching the pure brilliant colors often seen in these coronas.

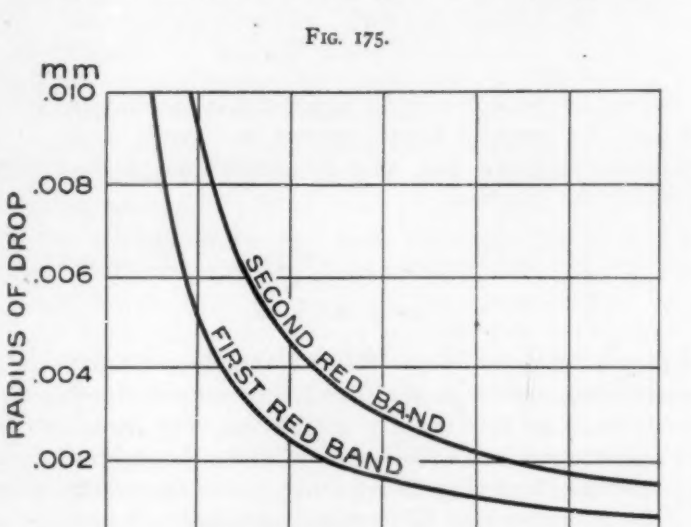
Presumably, therefore, the brilliant coronas of high clouds are due to very small undercooled water droplets of approximately uniform size, and not, as has generally been supposed, to ice needles.

Iridescent Clouds.—Thin and perhaps slowly evaporating

[199] *l. c.*

cirro-stratus and cirro-cumulus clouds occasionally develop numerous iridescent borders and patches of irregular shape, especially of red and green, at various distances from the sun up to 30° or more. A brilliantly colored iridescent cloud of considerable area is justly regarded as one of the most beautiful of sky phenomena, but one of which until recently there was no satisfactory explanation. Simpson,[200] however, has shown that the colored patches in question, presumably, are only fragments of coronas formed by exceedingly small droplets of very approximately

FIG. 175.



Relation between size of drop and size of corona.

uniform size. The relation between the radius of droplet and angular distances from the centre to the first and second red bands is shown in Fig. 175, also copied from the paper cited, from which it appears that coronas of the requisite size may occur, and, therefore, that the assumption that iridescent clouds are only fragments of unusually large and exceptionally brilliant coronas presumably is correct.

Bishop's Ring.—After the eruption of Krakatoa in 1883, of

[200] *l. c.*

Mont Pelé in 1902, and of Katmai in 1912, a faint reddish-brown corona was often seen, under favorable circumstances, around the sun. This is known as Bishop's ring, after Mr. Bishop of Honolulu, who first described it.

The width of this ring, as seen after the eruption of Krakatoa, was about 10°, and the distance from the sun to its outer edge, that is, to the first minimum, 22° to 23°. Substituting this value of the angular radius of the first minimum in the equation, explained above,

$$\sin\theta = (n + 0.22)\frac{\lambda}{2a}$$

and letting $\lambda = .000571$ mm., it appears that the diameter of the dust particles assumed either spheres or circular disks of approximately uniform size, that produced this peculiar corona as given by the equation,

$$2a = 1.22\,\frac{.000571\text{mm.}}{\sin 22^\circ\,30'}$$

$$= .00182 \text{ mm. about.}$$

Glory or Brocken-Bow.—When favorably situated, one occasionally may see rings of colored light around the shadow of his own head as cast upon a neighboring fog bank or cloud. This phenomenon, to which several names have been given—glory, Brocken-bow, Brocken-spectre, mountain-spectre—is produced by the diffraction by particles comparatively near the surface of light reflected from deeper portions of the fog or cloud.

The reflected light obviously emerges in every direction, but the nearer one looks along the path of incidence the larger the ratio of illuminated to non-illuminated particles in his line of sight. Indeed, at any appreciable angle from this special direction a considerable proportion of the droplets in one's vision evidently must lie in the shadows of others nearer the surface. Hence, not only will the shadow of one's head be surrounded by the brightest reflected light, like the "heiligenschein" one may see around the shadow of his head on a bedewed lawn, but it will also be the centre of the brightest and only perceptible glory or reflection halo, and that for the simple reason that the more intense the initial light the more brilliant its diffraction effects.

CHAPTER VII.

PHENOMENA DUE TO SCATTERING: COLOR OF THE SKY.

THE color of the cloudless sky, though generally blue, may, according to circumstances, be anything within the range of the entire spectrum. At great altitudes the zenithal portions are distinctly violet, but at moderate elevations often a clear blue. With increase of the angular distance from the vertical, however, an admixture of white light soon becomes perceptible that often merges into a grayish horizon. Just after sunset and also before sunrise portions of the sky often are distinctly green, yellow, orange, or even dark red, according especially to location and to the humidity and dust content of the atmosphere. Hence, these colors and the general appearance of the sky have rightly been used immemorially as more or less trustworthy signs of the coming weather.

Many attempts have been made to account for the blue of the sky [201]—the other colors being comparatively ignored. Some have held that it is just the nature of the atmosphere, or of particles in it, to reflect the blue of sunlight and to transmit the other colors. But as they did not explain how the atmosphere, or these particles, happened to have such nature the mystery actually remained as profound as ever. Another interesting hypothesis, suggested by Leonardo da Vinci, was to the effect that the blue is the resultant of a mixture of more or less white light, reflected by the atmosphere, with the black of space. But the futility of this idea is immediately obvious from the fact that gray alone could be produced by any such mixture.

The first logical attempt to explain (as that term is now understood) why the sky is blue was made by Newton,[202] who supposed it to be due to the same sort of interference between the rays reflected from the front and rear surfaces of transparent objects (in this case minute water drops) that produce the colors of soap bubbles. In fact, he thought that the "blue of the first order," the blue nearest the black central spot of the "Newton's rings," is of the same color as the blue of the sky, and that they were produced in the same way. This explanation, though erroneous, and based only on analogy, was accepted without modi-

[201] See summary and bibliography by Dorsey, *Monthly Weather Review* 28, p. 382, 1900.

[202] *Optics*, book ii.

fication for nearly 175 years. At about the end of this period, however, Clausius [203] demonstrated analytically that a cloud of droplets of the small size assumed by Newton would cause the stars and other celestial objects to appear enormously magnified. He, therefore, modified Newton's theory by assuming that the droplets are larger but vesicular with very thin walls. In this way the magnification trouble is avoided, but the theory is not improved. First, because water droplets are not hollow; and, second, because, as shown by Brücke,[204] the color of the sky differs radically from the blue of the "first order."

Although the above appears to have been the first serious criticism of the Newtonian theory of sky colors, observational and experimental data sufficient to render it untenable had long been known. This consisted of (*a*) Arago's [205] discovery in 1811 that sky light is partially polarized and that this polarization is a maximum along a circle about 90° from the sun; and (*b*) Brewster's [206] discovery, shortly thereafter, that polarization by reflection is a maximum when the tangent of the angle of incidence is equal to the refractive index of the reflector divided by that of the adjacent medium.

If, then, sky light is the result of simple reflection, the angle of polarization (angle of incidence corresponding to maximum polarization) of the reflecting medium must be 45°—since the arc of maximum polarization is 90° from the sun. But the angle of polarization of water in air is about 74°. Hence, the color of the sky cannot be due to reflection from water droplets, as Newton and many others assumed.

The real origin of the blue of the sky, scattering of light by particles far too small to reflect specularly, appears to have been first indicated by Brücke's [207] experiments, which showed (*a*) that a transparent medium, rendered turbulent by sufficiently small particles, appears blue when illuminated with white light; and (*b*) that objects may be seen through such medium clearly and distinctly. A few years later, Tyndall [208] made a large number of experiments on the action of chemically formed "clouds"

[203] *Crell's Jr.*, **34**, p. 122, 1847; p. 185, 1848; *Pogg. Ann.*, **72**, p. 294, 1847.
[204] *Pogg. Ann.*, **88**, p. 363, 1853.
[205] *Oeuvres*, **7**, p. 394, and p. 430.
[206] *Phil. Trans.*, **33**, p. 125, 1815.
[207] *l. c.*
[208] *Phil. Mag.*, **37**, p. 384, 1869; **38**, p. 156, 1869.

on incident white light, and found that not only did they scatter blue light when their particles were very small, but also that this light was completely polarized at right angles to the incident beam. Here, then, was the experimental solution of the problem of the blue of the sky and its polarization. About two years later, Lord Rayleigh [209] supplied the necessary theory, and thus, at last, one of the oldest and most difficult of the many problems of meteorological optics became completely solved. In a later paper Lord Rayleigh [210] showed that in the absence of dust of all kinds "the light scattered from the molecules [of air] would suffice to give us a blue sky, not so very greatly darker than that actually enjoyed." And still later, King [211] concluded that "The analysis of the present paper seems to support the view that at levels about Mount Wilson [1730 metres] molecular scattering is sufficient to account completely both for attenuation of solar radiation and for the intensity and quality of sky radiation." However, whether the scattering be by fine dust or by individual molecules the theory is the same, and, as developed in Rayleigh's first paper, substantially as follows:

Let a beam of light of wave-length λ be incident, say, to be definite, from the zenith. There will be little or no scattering from that portion of the beam in free ether, as is obvious from the facts (*a*) that extremely distant stars are still visible, and (*b*) that interstellar spaces are nearly black. From the portion in the atmosphere, however, there is abundant lateral scattering by the innumerable particles of dust and molecules of air, each of which is optically denser than the ether and so small in comparison to λ^3 that the applied force is practically constant throughout its volume. Each such particle merely increases local inertia of the ether, and, thereby, since the rigidity is not affected, correspondingly reduces the amplitude of a passing light wave. If, then, a force should be applied to each particle, such as to counterbalance the increasing inertia, the light would pass on exactly as in empty space and, therefore, without scattering. On the other hand, precisely the same force, but reversed in direction, if acting alone on free ether would produce the same effect that the disturbing particle produces. This force obvi-

[209] *Phil. Mag.*, **41**, pp. 107, 274, 447, 1871; **12**, p. 81, 1881.
[210] *Phil. Mag.*, **47**, p. 375, 1899.
[211] *Phil. Trans., A.*, **212**, 375, 1913.

ously must have the same period and direction as the undisturbed luminous vibrations and be proportional to the difference in optical density between the particle and the ether.

The only factors that conceivably can affect the ratio of the amplitude of scattered to incident light are: direction, or, rather, angle between directions of force and point of observation; ratio between the optical densities of the disturbing particle and the ether; volume of particle; distance from particle; wave-length; and velocity of light. Hence, in comparing the extents to which lights of different colors are scattered, the first two factors may be neglected, since they apply in equal measure to all. Furthermore, as the ratio in question, like all ratios, is a mere number and, therefore, dimensionless, the last factor must be omitted, since it and it alone involves time. There remain, then, only the volume of the particle, distance from it, and the wave-length to consider. But from the dynamics of the problem it appears that the ratio of the two amplitudes must vary directly as the volume of the particle and inversely as the distance from it. That is,

$$N = \frac{L^3}{L} f(\lambda),$$

in which N is some number, L a unit of length, and $f(\lambda)$ that function of λ that renders the equation dimensionless. Hence,

$$f(\lambda) = \lambda^{-1},$$

and, therefore, the ratio of the two intensities is proportional to λ^{-4}. Obviously, then, light from a serene sky should belong essentially to the blue or short wave-length end of the spectrum.

If, as commonly expressed, the displacement in the incident wave is $A \cos\left(\frac{2\pi\, v\, t}{\lambda}\right)$, in which A is the amplitude; v the velocity of light, λ the wave-length; and t the time since any convenient interval when the displacement was A, then the corresponding acceleration is

$$\frac{d^2}{dt^2} A \cos \frac{2\pi\, v\, t}{\lambda} = - A \left(\frac{2\pi}{\lambda}\, v\right)^2 \cos \frac{2\pi}{\lambda}\, vt.$$

Hence, the force that would have to be applied to a sufficiently minute particle in order that the wave might pass over it undisturbed is

$$- (D' - D)\, T\, A \left(\frac{2\pi\, v}{\lambda}\right)^2 \cos \frac{2\pi}{\lambda}\, vt.$$

in which D' and D are the optical densities of the particle and ether, respectively; and T the volume of the particle. And this, as explained, is also the expression for the force which, if operating alone on the ether, would produce the same light effects that actually are induced by the particle in question.

Now it has been shown by Stokes,[212] and also by Lord Rayleigh,[213] that the displacement X produced by the force $F \cos \frac{2\pi\, vt}{\lambda}$ is given by the expression,

$$X = \frac{F \sin\alpha}{4\pi v^2 D r} \cos \frac{2\pi}{\lambda} (vt - r)$$

in which α is the angle between the direction of the force and the radius vector, r, that connects the centre of the force with the point at which the displacement is observed.

On substituting for the force $F \cos \frac{2\pi\, v\, t}{\lambda}$ its value, one finds that

$$X = A \frac{D' - D}{D} \frac{\pi}{r} \frac{T}{\lambda^2} \sin\alpha \cos \frac{2\pi}{\lambda} (v\, t - r)$$

Hence, the intensity of the light scattered by a single particle is

$$A^2 \left(\frac{D' - D}{D}\right)^2 \frac{\pi^2 T^2}{r^2 \lambda^4} \sin^2\alpha.$$

and for a cloud

$$A^2 \left(\frac{D' - D}{D}\right) \frac{\pi^2 \sin^2\alpha}{\lambda^4} \Sigma \frac{T^2}{r^2},$$

in which $\Sigma \frac{T^2}{r^2}$ is the sum of the values of $\frac{T^2}{r^2}$ for all the particles in the line of sight, or

$$A^2 \left(\frac{D' - D}{D}\right)^2 \frac{\pi^2 \sin^2\alpha}{\lambda^4} N \left(\frac{T}{r}\right)^2_m$$

in which N is the total number of particles in the line of sight, and $\left(\frac{T}{r}\right)^2_m$ the mean of the several values of $\left(\frac{T}{r}\right)^2$.

The above equations are based on the assumption that the displacements in the incident wave are all in the same plane—that the incident light is plane polarized. If, however, they lie in parallel planes, passing through the axis of propagation, that is, if the incident light is unpolarized, we may resolve each dis-

[212] *Camb. Phil. Trans.*, **9**, p. 1, 1849; *Math. and Phys. Papers II*, pp. 243-328.

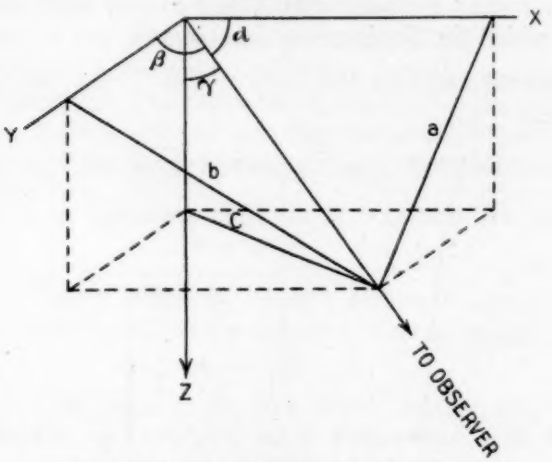
[213] *Phil. Mag.*, **41**, p. 107, 1871.

placement, always normal to the line of travel, into two components at right angles to each other and obtain their joint effect in any given direction. Let the line from the centre of the force to the point of observation make the angles α, β, and γ with these components and the direction of travel, respectively. Then, since $\sin^2\alpha + \sin^2\beta = 1 + \cos^2$, (see Fig. 176), the intensity of the scattered light at the angle γ from the direction of travel of a non-polarized beam is,

$$A^2 \left(\frac{D' - D}{D}\right)^2 \frac{\pi^2 (1 + \cos^2\gamma)}{\lambda^4} N \left(\frac{T}{r}\right)^2_m$$

According to this equation, the maximum amount of scattered

FIG. 176.



Intensity of scattered light in a given direction.

light is along the path—forward and back—of the incident beam, and least at right angles to it. Also, the intensity is directly proportional to the square of the volume of the disturbing particle, provided it is sufficiently small.

The effect of the size of particle on scattering as it approaches λ^3 is not well known. However, Lord Rayleigh [214] has shown that the intensity of the light scattered by relatively large spherical particles varies as the inverse 8th power of the wave-length.

Extinction Coefficient.—The intensity or brightness of solar or other radiation is decreased with increase of air path by (*a*)

[214] *Phil. Mag.*, **12**, p. 81, 1881.

scattering, (*b*) selective absorption, (*c*) diffraction, (*d*) reflection, and (*e*) refraction. When the sky is clear, however, only (*a*) is particularly effective in the visual region, but here it is quite effective, since each disturbing particle evidently scatters energy from incident radiation in proportion to the expression,

$$\frac{\pi^2 T^2}{r^2 \lambda^4}\left(\frac{D' - D}{D}\right)^2 \int_0^{\pi} \sin^2\alpha 2\pi r^2 \sin\alpha d\alpha = \frac{8\pi^3 T^2}{3 \lambda^4}\left(\frac{D' - D}{D}\right)^2.$$

Let E be the energy delivered per unit cross-section of the incident beam in any interval of time, and let n be the number of disturbing particles (all alike) per unit volume. Then the energy gain (negative) during the same time per unit cross-section, and penetration, dx, is given by the equation,

$$dE = - E n dx \frac{8\pi^3 T^2}{3\lambda^4}\left(\frac{D' - D}{D}\right)^2.$$

If, then, E_0 is the energy in the beam before any scattering took place, and E the energy remaining after penetrating the distance x into the turbulent medium in question,

$$E = E_0 e^{-\epsilon x}$$

in which the extinction coefficient

$$\epsilon = \frac{8\pi n T^2}{3 \lambda^4}\left(\frac{D' - D}{D}\right)^2.$$

But $D = \mu^2$ (from the equations, $v = \sqrt{\frac{\text{Elasticity}}{\text{Density}}} = \frac{1}{\mu}$, density of ether = 1, μ for either = 1), μ = refractive index. Also, in the case of scattering by air molecules, or by any small particles in a medium whose refractive idex is 1,

$$n T \left(\frac{D' - D}{D}\right) = D'' - 1$$

in which D'' = average optical density of the turbulent space. If μ is the refractive index of the medium, air, say,

$$n T \left(\frac{D' - D}{D}\right) = \mu^2 - 1 = (\mu + 1)(\mu - 1) = 2(\mu - 1), \text{ nearly,}$$

since μ differs but little from unity.

Hence, substituting,

$$\epsilon = \frac{32 \pi^3}{3 n \lambda^4}(\mu - 1)^2, \text{ approximately.}$$

Clearly, then, the intensity of atmospheric and dust haze rapidly decreases with increase of wave-length, a fact that jus-

tifies the use, on aeroplanes, for instance, of "haze cutters" (filters that transmit only the longer waves) for both visual and photographic work. A fog haze, however, cannot be much cut. This is because the extinction it produces, being due, owing to the relatively large size of the fog droplets, chiefly to diffraction and reflection, is nearly equally effective for all colors.

The scattering of light by the molecules of the atmosphere and the suspended fine dust particles decrease the intensity of both the direct insolation and the scattered radiation, but at the same time gives to all portions of the sky, other than that occupied by the sun, a luminosity that otherwise would not exist—without scattering there would be no sky light at all. The residual sunlight and the total sky light vary greatly with time of day, latitude, altitude, season, weather, and dustiness of the atmosphere. Kimball,[215] for instance, finds "that photometric measurements made at Mount Weather, Va. [Lat. 39° 4′ N., Long. 77° 54′ W., altitude 526 m.], show that with a clear sky the total mid-day illumination on a horizontal surface varied from 10,000 foot-candles in June to 3600 foot-candles in January. It is less than the direct solar illumination on a normal surface from September to February, inclusive, but exceeds the latter from May to August, inclusive, for a period of from four to eight hours in the middle of the day.

"The illumination on a horizontal surface from a completely overcast sky may be half as great as the total illumination with a clear sky, and is frequently one-third as great. On the other hand, during severe thunderstorms at noon in midsummer, the illumination may be reduced to less than 1 per cent. of the illumination with a clear sky.

"The ratio of sky-light illumination to total illumination on a horizontal surface at noon in midsummer varies from one-third to one-tenth. In midwinter it varies from one-half to one-fifth.

"When the sky is clear, the twilight illumination on a horizontal surface falls to 1 foot-candle about half an hour after sunset, or when the sun is about 6° below the horizon."

Prevailing Color.—If I_0 is the initial intensity and I_1 the remaining intensity after penetrating the uniformly turbulent medium, the distance x, then,

$$I_1 = I_0 E^{-\frac{kx}{4}}$$

[215] *Monthly Weather Review*, **42**, p. 650, 1914.

where

$$k = \frac{32\,\pi^3}{3\,n}\,(\mu - 1)^2.$$

This residual light, in turn, is scattered, and if I_2 is the intensity of the light scattered by a single particle at the angle α from the direction of displacement

$$I_2 = I_1 \frac{k'}{\lambda^4}$$

where

$$k' = \frac{\pi^2\,T^2}{r^2}\left(\frac{D' - D}{D}\right)^2 \sin^2\alpha,$$

and

$$I_2 = I_0 \frac{k'}{\lambda^4}\, e^{-\frac{kx}{\lambda^4}}.$$

Hence, the ratio of intensity received to initial intensity, or I_2/I_0, is small both for very long and very short wave-lengths. Its maximum value occurs at $\lambda^4 m = kx$, where

$$\left(\frac{I_2}{I_0}\right) = \left(\frac{I_2}{I_0}\right)_m \left(\frac{\lambda_m}{\lambda}\right)^4 e^{1-\left(\frac{\lambda m}{\lambda}\right)^4},$$

or, if I_0 is uniform throughout the spectrum,

$$I_2 = I_m \left(\frac{\lambda_m}{\lambda}\right)^4 e^{1-\left(\frac{\lambda m}{\lambda}\right)^4}.$$

According to Abbot, Fowle, and Aldrich,[216] the mean energy intensities of I_0, in arbitrary units, are:

$\lambda =$	0.39	0.42	0.43	0.45	0.47	0.50	0.55	0.60	0.70
$I_0 =$	3614	5251	5321	6027	6240	6062	5623	5042	3644

The luminous intensities would show greater contrasts, since the eye is more sensitive to the mid-region of the visible spectrum than to either end.

From these values of I_0, and the equation for the intensity of I_2, it can be shown that the prevailing color of the clear sky, except when the sun is on or below the horizon, is neither violet nor red, but some intermediate color, generally blue, as we know by observation.

Twilight Colors.—As the sun sinks to and below the horizon during clear weather, a number of color changes occur over large portions of the sky, especially the eastern and western. The phenomena that actually occur vary greatly, but the following

[216] *Annals Astrophys. Obsy.*, Smithsonian Institution, 3, p. 197, 1913.

may be regarded as typical, especially for arid and semi-arid regions:

(*a*) A whitish, yellowish, or even bronze glow of 5° or 6° radius that concentrically encircles the sun as it approaches the horizon, and whose upper segment remains visible for perhaps 20 minutes after sundown.

The chief contributing factors to this glow appear to be (1) scattering, which is a maximum in the direction, forward and back, of the initial radiation, and (2) diffraction by the dust particles of the lower atmosphere. In both cases blue and violet are practically excluded, owing to the very long air paths.

(*b*) A grayish blue circle that rises above the eastern horizon as the sun sinks below the western. This is merely the shadow of the earth.

(*c*) A purplish arch that rests on the earth shadow and gradually merges into the blue of the sky at a distance of perhaps 10°, and also fades away as the arch rises.

Obviously, any direct sunlight in the lower dusty atmosphere to the east must have penetrated long distances through the denser air, and thus have become prevailing red, while that reaching the higher atmosphere is still rich in blue and violet. Hence, the observer sees red light scattered from the first of these layers and blue to violet from the other, and thereby gets the effect of the superposition of the opposite ends of the visible spectrum, that is, purple. The effect is most pronounced when the luminous layers are seen more or less "end on." Hence, the light is brightest at the border of the earth shadow. The fact that the red component of the purple is from the lower atmosphere and the others from the higher is evident from the bluish crepuscular rays that often radiate, apparently, from the antisolar point—shadow streaks cast through the lower dust-laden air by western clouds or mountain peaks, often below the horizon.

(*d*) A bright segment only a few degrees deep but many in extent that rests on the western horizon just after sundown. The lowest portion often is red and the upper yellowish. A product essentially of scattered light by the lower and dustier portions of the atmosphere, where the light before being scattered is already reduced essentially to the colors seen.

(*e*) A purple glow covering much of the western sky, reaching its maximum intensity when the sun is about 4° below the

horizon and disappearing when it is about 6° below. The explanation of this purple glow in the western sky presumably is the same as that in the eastern sky as given above under (*c*). The crepuscular rays of this region, apparently radiating from the sun, often are greenish-blue.

(*f*) A faint purple glow covering the entire sky when the sun is 6° or more below the horizon, and gradually disappearing in the west when the sun is 16° to 18° below the horizon. This appears to be due to secondary scattering of light from the illuminated atmosphere far to the west.

Duration of Astronomical Twilight. (Interval Between Times When the Upper Edge of the Sun is on and the True Position of Its Centre 18° Below the Horizon.)

Date		North latitude														
		0°	10°	20°	25°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
		h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.	h.m.
January	I	1 14	1 15	1 18	1 21	1 26	1 28	1 29	1 31	1 34	1 37	1 41	1 45	1 49	1 53	1 59
	II	1 14	1 14	1 18	1 21	1 25	1 27	1 29	1 31	1 33	1 36	1 39	1 43	1 37	1 52	1 57
	21	1 13	1 13	1 17	1 20	1 23	1 25	1 28	1 30	1 32	1 34	1 38	1 41	1 45	1 49	1 54
February	I	1 12	1 12	1 15	1 18	1 22	1 24	1 26	1 28	1 30	1 33	1 36	1 39	1 43	1 47	1 52
	II	1 11	1 12	1 14	1 17	1 21	1 23	1 25	1 27	1 29	1 32	1 34	1 37	1 41	1 45	1 49
	21	1 10	1 11	1 13	1 16	1 20	1 22	1 24	1 26	1 28	1 31	1 33	1 36	1 40	1 44	1 48
March	I	1 10	1 11	1 13	1 16	1 20	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
	II	1 09	1 10	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
	21	1 09	1 10	1 13	1 16	1 20	1 22	1 24	1 26	1 29	1 31	1 34	1 37	1 41	1 45	1 50
April	I	1 09	1 11	1 14	1 17	1 21	1 23	1 25	1 27	1 30	1 33	1 36	1 40	1 44	1 49	1 54
	II	1 10	1 11	1 15	1 18	1 22	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 54	2 00
	21	1 11	1 12	1 16	1 20	1 24	1 27	1 29	1 32	1 36	1 39	1 43	1 48	1 54	2 01	2 08
May	I	1 12	1 13	1 18	1 22	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 54	2 01	2 10	2 20
	II	1 13	1 14	1 19	1 24	1 30	1 33	1 36	1 40	1 43	1 48	1 54	2 01	2 10	2 20	2 35
	21	1 13	1 15	1 21	1 26	1 32	1 36	1 39	1 43	1 48	1 54	2 01	2 10	2 20	2 35	2 58
June	I	1 14	1 16	1 23	1 28	1 35	1 38	1 41	1 46	1 52	1 59	2 07	2 18	2 31	2 54	....
	II	1 15	1 17	1 24	1 29	1 36	1 40	1 44	1 49	1 55	2 02	2 12	2 23	2 40	3 11	....
	21	1 15	1 18	1 24	1 29	1 37	1 41	1 45	1 50	1 56	2 03	2 13	2 25	2 44	3 19	....
July	I	1 15	1 17	1 24	1 29	1 36	1 40	1 44	1 49	1 55	2 02	2 12	2 23	2 40	3 10	....
	II	1 14	1 16	1 23	1 28	1 35	1 38	1 41	1 46	1 52	1 59	2 07	2 18	2 31	2 54	....
	21	1 13	1 15	1 21	1 26	1 32	1 36	1 39	1 43	1 48	1 54	2 01	2 10	2 21	2 36	3 00
August	I	1 13	1 14	1 19	1 24	1 30	1 33	1 36	1 40	1 44	1 48	1 54	2 02	2 10	2 20	2 35
	II	1 12	1 13	1 18	1 22	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 54	2 01	2 10	2 20
	21	1 11	1 12	1 16	1 20	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 54	2 01	2 09
September	I	1 10	1 11	1 14	1 18	1 22	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 53	2 00
	II	1 09	1 11	1 13	1 17	1 21	1 23	1 25	1 27	1 30	1 33	1 36	1 39	1 44	1 49	1 54
	21	1 09	1 10	1 13	1 16	1 20	1 22	1 24	1 26	1 29	1 31	1 34	1 37	1 41	1 45	1 50
October	I	1 09	1 10	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
	II	1 10	1 11	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
	21	1 10	1 11	1 13	1 16	1 20	1 22	1 24	1 26	1 28	1 31	1 33	1 36	1 40	1 44	1 48
November	I	1 11	1 12	1 14	1 17	1 21	1 23	1 25	1 27	1 29	1 32	1 34	1 38	1 41	1 46	1 49
	II	1 12	1 12	1 16	1 18	1 22	1 24	1 26	1 28	1 30	1 33	1 36	1 40	1 43	1 47	1 52
	21	1 13	1 13	1 17	1 20	1 24	1 26	1 28	1 30	1 32	1 35	1 38	1 42	1 46	1 49	1 55
December	I	1 14	1 14	1 18	1 21	1 25	1 27	1 29	1 31	1 33	1 36	1 40	1 44	1 47	1 52	1 57
	II	1 14	1 15	1 18	1 22	1 26	1 28	1 30	1 32	1 34	1 37	1 41	1 45	1 49	1 53	1 59
	21	1 15	1 16	1 19	1 22	1 26	1 28	1 30	1 32	1 35	1 38	1 41	1 45	1 49	1 54	1 59

Duration of Civil Twilight. (*Interval Between Times When the Upper Edge of the Sun is on and the True Position of Its Centre 6° Below the Horizon.*)

Date		North latitude														
		0°	10°	20°	25°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
		m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.
January	I	22	22	24	25	27	27	27	28	29	30	32	33	34	36	39
	II	22	22	24	25	26	27	28	28	29	30	31	32	33	35	38
	21	22	22	23	24	26	26	27	27	28	29	30	32	33	34	37
February	I	22	22	23	24	25	26	27	27	27	28	29	31	32	34	35
	II	22	22	22	23	25	26	26	27	27	28	29	31	32	34	35
	21	21	22	22	23	24	25	25	26	27	28	28	29	30	32	33
March	I	21	22	22	23	24	24	25	26	27	28	28	29	30	31	33
	II	21	21	22	23	24	24	25	26	26	27	27	28	30	31	33
	21	21	21	22	23	24	24	25	26	26	27	27	28	30	31	33
April	I	21	21	22	23	24	25	25	26	27	28	28	29	30	32	33
	II	21	22	22	23	24	25	26	26	27	28	28	29	31	32	34
	21	22	22	22	23	25	25	26	27	28	28	29	30	32	34	35
May	I	22	22	23	24	25	26	27	28	28	29	30	32	33	35	36
	II	22	22	23	24	26	27	28	29	29	30	31	33	35	36	39
	21	22	22	24	25	27	28	28	29	30	31	33	35	36	38	41
June	I	22	22	24	25	27	28	28	29	31	32	34	36	37	40	43
	II	22	23	24	26	28	28	29	30	31	33	34	36	38	41	44
	21	22	23	25	26	28	29	29	30	31	33	34	36	38	42	44
July	I	22	23	24	26	28	28	29	30	31	33	34	36	38	41	44
	II	22	22	24	25	27	28	28	29	31	32	34	36	37	40	43
	21	22	22	24	25	27	28	28	29	30	31	33	35	36	38	41
August	I	22	22	23	24	26	27	28	29	29	30	31	33	35	36	39
	II	22	22	23	24	25	26	27	28	28	29	30	32	33	35	36
	21	22	22	22	23	25	25	26	27	28	28	29	30	32	34	35
September	I	21	22	22	23	24	25	26	26	27	28	28	29	31	32	34
	II	21	21	22	23	24	25	25	26	27	28	28	29	30	31	33
	21	21	21	22	23	24	24	25	26	27	27	27	28	30	31	33
October	I	21	21	22	23	24	24	25	26	26	27	27	29	30	31	32
	II	21	22	22	23	24	24	25	26	27	28	28	29	30	31	33
	21	21	22	22	23	24	25	25	26	27	28	28	29	30	32	33
November	I	22	22	22	23	25	25	26	27	28	28	29	30	31	33	34
	II	22	22	23	24	25	26	27	28	28	29	30	31	32	33	35
	21	22	22	23	24	26	26	27	28	28	29	30	32	33	34	37
December	I	22	22	24	25	26	27	28	28	29	30	31	33	34	35	38
	II	22	22	24	25	27	27	28	28	29	30	32	33	34	36	39
	21	22	23	24	25	27	27	28	28	29	31	32	33	34	37	39

Relative Illumination Intensities.

Source of illumination	Intensity	Ratio to zenithal full moon
	Foot-candles	
Zenithal sun	9,600.0	465,000.0
Twilight at sunset or sunrise	33.0	1,598.0
Twilight centre of sun 1° below horizon	30.0	1,453.0
Twilight centre of sun 2° below horizon	15.0	727.0
Twilight centre of sun 3° below horizon	7.4	358.0
Twilight centre of sun 4° below horizon	3.1	150.0
Twilight centre of sun 5° below horizon	1.1	53.0
Twilight centre of sun 6° below horizon (End of civil)	0.40	19.0
Twilight centre of sun 7° below horizon	0.10	5.0
Twilight centre of sun 8° below horizon	0.04	2.0
Twilight centre of sun 8°40′ below horizon	0.02	1.0
Zenithal full moon	0.02	1.0
Twilight centre of sun 9° below horizon	0.015	0.75
Twilight centre of sun 10° below horizon	0.008	0.40
Starlight	0.00008	0.004

The foregoing descriptions, which, of course, apply equally to dawn, are by no means universally applicable. Indeed, the sky very commonly is greenish instead of purple, probably when the atmosphere is but moderately dust-laden. Furthermore, the explanations are only qualitative. A rigid analysis, even if the distribution of the atmosphere and its dust and moisture content were known—which they are not, nor are they constant—would be at least difficult and tedious.

Duration of Twilight.—The duration of twilight, whether civil, that is, the time after sunset or before sunrise during which there is sufficient light for outdoor occupations, or astronomical, the time until or after complete darkness, varies with the amount of cloudiness and inclination of the ecliptic to the horizon. In the case of clear skies, civil twilight ends, or begins, when the true position of the sun (centre) is about 6° below the horizon, and astronomical twilight when it is about 18° below.

The tables of twilight duration (pages 667 and 668) were computed by Kimball [217] from the equation,

$$h = \frac{\sin \alpha - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

in which h is the sun's hour angle from the meridian, α the sun's altitude (negative below the horizon), δ the solar declination, and ϕ the latitude.

[217] *Monthly Weather Review*, **44**, p. 614, 1916.

Twilight Illumination.—The brightness of twilight changes slowly or rapidly, according as the sun is less or more, respectively, than about 4° below the horizon. The following table, based on photometric measurements by Kimball and Thiessen,[218] gives the approximate value of a number of clear-sky, twilight and other natural illumination intensities on a fully exposed horizontal surface.

CHAPTER VIII.

PHENOMENA DUE TO SCATTERING: SKY POLARIZATION

THE polarization of sky light, discovered in 1811 by Arago,[219] often is more or less modified by specular reflection from relatively large particles—cloud droplets, coarse dust, etc.—but in general it results from the combination of primarily and secondarily scattered radiation.

Condition of Primarily Scattered Light.—As explained by Lord Rayleigh,[220] the light scattered from an incident beam by a gas molecule or other sufficiently small object is symmetrically distributed about the line of enforced motion of that particle as an axis, and completely polarized in the plane at right angles to this line. This follows directly from the fact that plane polarized light is merely light whose vibrations are all normal to the same plane—the plane of polarization. If, then, the incident beam is non-polarized, ordinary sunlight, for instance, the scattered light, therefore, will be completely polarized at right angles to the direction of incidence. and partially polarized in other directions. And, since the plane of polarization is fixed by the sun, observer, and point observed, it follows from Fig. 176 that the ratio

$$\frac{\text{polarized light}}{\text{total light}} = \frac{\sin^2 \gamma}{1 + \cos^2 \gamma},$$

where γ = the angular distance of the point observed from the sun. That is, the polarization increases from zero in the direction both of the sun and the antisolar point to a maximum (complete) midway between them, or normal to the incident rays.

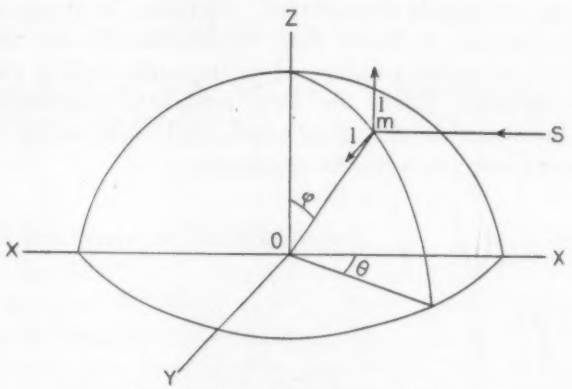
Condition of Secondarily Scattered Light.—While primary scattering of ordinary light by gas molecules and fine dust par-

[218] *Monthly Weather Review*, **44**, p. 614, 1916.
[219] *Astronomie Populaire*, **2**, p. 99.
[220] *Phil. Mag.*, **41**, p. 107, 1871.

ticles accounts for a large part of the observed polarization and other phenomena of sky light, the non-polarized light that always exists in a greater or less amount at 90° from the sun; the luminosity—partially polarized—of shaded air masses; and the existence of neutral points (small regions whose light is not polarized) are all due, as Soret [221] has shown, to secondary scattering. Tertiary and indefinitely higher scattering obviously also exist, but their effects are too small to justify consideration.

To determine the nature and magnitude of secondary scattering, let O (Fig. 177) be the position of a particle shielded from direct insolation but otherwise exposed, and consider its effect on the total incoming sky light. Let the sun be on the horizon;

FIG. 177.



Intensity of secondarily scattered light.

let OX be parallel to the solar radiation, OZ vertical, and OY normal to the plane ZX; let m be any particle a unit distance from O; and let Om make the angle ϕ with the vertical, and its projection on the plane XY the angle θ with OX.

As the solar rays are non-polarized they may be treated as consisting of two parts of equal amplitude, l, say, polarized at right angles to each other. For convenience, let the displacements be parallel to OZ and OY, corresponding to polarization in the horizontal and vertical planes, respectively.

On resolving the vertical amplitude into two components, one normal, the other parallel, to Om and the former (which alone is

[221] *Archives de Sci. Phys. et Nat.*, 20, p. 429, 1888.

operative on the particle at O) in turn into components parallel to the X, Y, and Z axes, respectively, one finds that,

$$l'_X = -l \sin\phi \cos\phi \cos\theta$$
$$l'_Y = -l \sin\phi \cos\phi \sin\theta$$
$$l'_Z = l \sin^2\phi$$

Similarly, on resolving the horizontal amplitude into components normal and parallel to the plane OZm, and these in turn parallel to the three axes, one obtains,

$$l''_X = l \cos^2\phi \sin\theta \cos\theta - l \sin\theta \cos\theta = -l \sin^2\phi \sin\theta \cos\theta$$
$$l''_Y = l \cos^2\phi \sin^2\theta + l \cos^2\theta$$
$$l''_Z = -l \sin\phi \cos\phi \sin\theta$$

As a crude first approximation let the distribution of the atmosphere about O be equal in all upward directions and assume all parts to be equally illuminated. Further, let a per unit area be the number of particles that, if distributed over the hemispherical shell, would produce at O the same optical effect that actually obtains. Then, the total intensity components at O (found by squaring the amplitudes and integrating over the hemisphere) are given by the equations,

$$I_X = 2\,a\,l^2 \int_0^{\frac{\pi}{2}} \int_0^{\pi} (\sin^2\phi \cos^2\phi \cos^2\theta + \sin^4\phi \sin^2\theta \cos^2\theta) \sin\phi \, d\phi \, d\theta$$

$$I_Y = 2al^2 \int_0^{\frac{\pi}{2}} \int_0^{\pi} (\sin^2\phi \cos^2\phi \sin^2\theta + \cos^4\phi \sin^4\theta + 2\cos^2\phi \sin^2\theta \cos^2\theta + \cos^4\theta) \sin\phi \, d\phi \, d\theta$$

$$I_Z = 2\,a\,l^2 \int_0^{\frac{\pi}{2}} \int_0^{\pi} (\sin^4\phi + \sin^2\phi \cos^2\phi \sin^2\theta) \sin\phi \, d\phi \, d\theta$$

Or,

$$I_X = 2\pi\, a\, l^2 \times 2/15$$
$$I_Y = 2\pi\, a\, l^2 \times 3/5$$
$$I_Z = 2\pi\, a\, l^2 \times 3/5$$

The intensity components of secondary diffusion at the centre of a sphere of uniformly distributed particles would be just twice the above, which, as stated, applies to the centre of a hemisphere.

Since there is an appreciable amplitude along all three of the rectangular axes, it follows that secondary scattering sends

more or less non-polarized light in all directions, and, therefore, prevents sky light from being completely polarized even at right angles to the direction of insolation—the direction of complete polarization by primary scattering.

The above assumption that the light-scattering particles are distributed equally along any upward radius from O obviously is not in close agreement with the actual distribution of the atmosphere and its dust content as visible from any given point in it. Let this distribution be $a(n+1) - a\,n \cos \phi$ particles per unit area of the hemisphere instead of a, as previously assumed. Then,

$$I_X = 2\pi a\, l^2 \left[2/15\,(n+1) - 1/16\,n \right]$$
$$I_Y = 2\pi a\, l^2 \left[3/5\,(n+1) - 17/48\,n \right]$$
$$I_Z = 2\pi a\, l^2 \left[3/5\,(n+1) - 5/24\,n \right].$$

If $n = 12$, that is, if the horizon is 13 times brighter than the zenith—a common condition,

$$I_X = 2\pi\, a\, l^2 \times 0.983$$
$$I_Y = 2\pi\, a\, l^2 \times 3.55$$
$$I_Z = 2\pi\, a\, l^2 \times 5.3$$

This distribution of intensities still gives non-polarized light in all directions. It also gives a preponderant amount of polarization, I_z, in the horizontal plane, which neutralizes at certain places the polarization in the vertical plane due to primary scattering.

The combination, then, of primarily and secondarily scattered light must produce a variety of polarization and other phenomena which necessarily vary with the altitude of the sun, dust content of the atmosphere, and state of the weather. Many observational studies have been made of sky polarization and the facts found to agree with the above theoretical considerations. The principal facts are:

(1) Part of the light from nearly all points in a clear sky is plane polarized, whatever the season, location, altitude of the sun, or other conditions.

(2) The polarized portion of sky light, in turn, is divisible into two parts: (*a*) the positive, due to the first or primary scattering, in which the plane of polarization (plane normal to the

vibrations) is given by the source (sun), point of observation, and eye of the observer; and (*b*) the negative, due to secondary scattering, in which the plane of polarization is normal to that of the primary, and, therefore, because of the ring-like distribution of the atmosphere about any point on the earth's surface, essentially horizontal.

(3) Generally speaking, the percentage of polarized light along any great circle connecting the sun and the antisolar point increases from zero near either to a maximum midway between them, which, in turn, increases with the altitude of the point in question.

(4) The point of absolute maximum polarization is in the solar vertical and ordinarily about 90°, as stated, from the sun.

(5) In general, the percentage of polarization decreases with the amount of light reflected through the sky, whether from the surface or from relatively large particles in suspension. It therefore decreases with (*a*) percentage of snow covering; (*b*) percentage of cloudiness; (*c*) dustiness, or anything that itself leads to an increase of dustiness, such as high winds, especially over arid regions, but everywhere during dry weather, strong vertical convection—hence, generally less during summer than winter—volcanic explosions of the Krakatoa type, etc.

(6) The percentage of polarization generally increases with the wave-length of the light examined.

(7) Even shaded masses of air, if exposed to sky radiation, emit perceptible amounts of polarized light.

(8) Three small regions of unpolarized light, Babinet's, Brewster's and Arago's neutral points, occur on the solar vertical; the first some 15° to 20° above the sun, the second about the same distance below it, and the third 20°, roughly, above the antisolar point.

(9) As the sun rises above or sinks below the horizon the antisolar distance of Arago's point increases from about 20° to, roughly, 23°; while the solar distance of Babinet's point decreases from a maximum of, approximately, 20° to, perhaps, 18°, for a solar depression of 5° or 6°, and to 0°, as does also Brewster's point, as the zenith is approached.

(10) When the upper atmosphere is greatly turbid, as it has often been after violent volcanic explosions, other neutral points, in addition to those above mentioned, are occasionally observed.

(To be Continued.)

THE HIDING POWER OF WHITE PIGMENTS AND PAINTS.*

BY

A. H. PFUND, Ph.D.

Associate Professor of Physics, Johns Hopkins University.

THE hiding power of a paint may be defined as that property of a paint which enables it to obliterate beyond recognition any background upon which it may be spread. In order to compare the hiding power of two white pigments, the almost universal custom is to rub down equal masses of the two samples with the same amount of linseed oil and ultramarine blue. The resultant pastes are spread side by side and the sample which has the paler tint is supposed to contain the pigment of greater hiding power. The idea is that the particles of white pigment hide the dark particles and hence give a light tint to the paste. As a result of the experiments to be described presently, it appears that the criterion, upon which the above tests are based, is incorrect.

The object of this investigation is to present a method which will yield numerical values of the true hiding power of white pigments and paints.[1] The basic idea underlying the discussion is this: granting that an infinitely thick layer of a paint will "hide" a given background completely, it is sought to find the thinnest layer which will hide the background as effectively as does the infinitely thick layer. Obviously, the thinner the layer of paint required, the greater is the hiding power of the paint. For white paints, the severest test met with in practice is a white wall with black lettering. It is desired to obliterate the lettering by covering the entire surface with successive coats of paint. These conditions are simulated in the instrument about to be described.

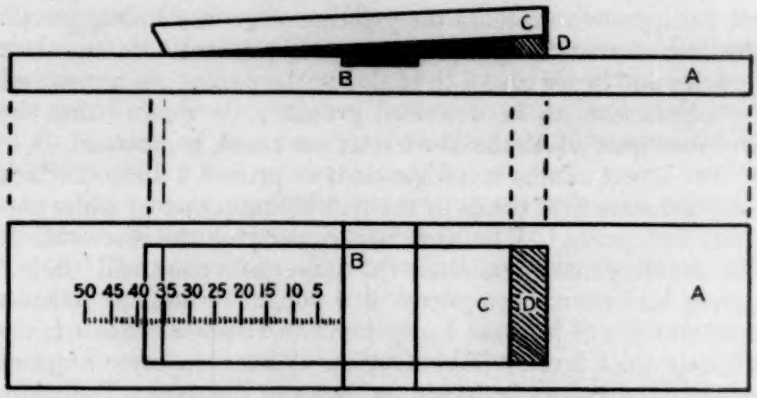
The form given the instrument is shown in Fig. 1. Here, A is a plate of glass $14 \times 5 \times .6$ cm. whose upper surface is optically flat. The lower surface is coated with black baking-enamel, which yields the desired black background. A transverse groove, B, about 2 mm. deep and 1 cm. wide, is cut in the upper surface and a millimetre scale is etched, as shown in the drawing. Resting upon plate A is plate C ($7 \times 3.5 \times .6$ cm.), whose lower surface is like-

* Communicated by the Author.

[1] First presented in a report to the New Jersey Zinc Co., Sept. 7, 1918.

wise optically flat. A strip of thin steel, *D*, 0.45 mm. thick, is attached to *C*, so that a wedge-shaped layer of white paint may be formed between the plates. This wedge terminates abruptly at the "infinitely thick" layer, *B*, and, so long as the hiding is not complete, the line of demarcation is visible. By sliding the wedge to the left it is finally impossible to see the edge. From a knowledge of the angle of the wedge and the reading on the scale, it is possible to calculate the thickness of this critical layer lying immediately above the edge *B*. Now, in advancing the plate *C* until the line of demarcation can no longer be seen, we have overdone it, so to speak. To correct this, we must reverse the motion of the wedge until the edge can just be distinguished over its entire length. The mean value of the reading corresponding, respec-

FIG. 1.



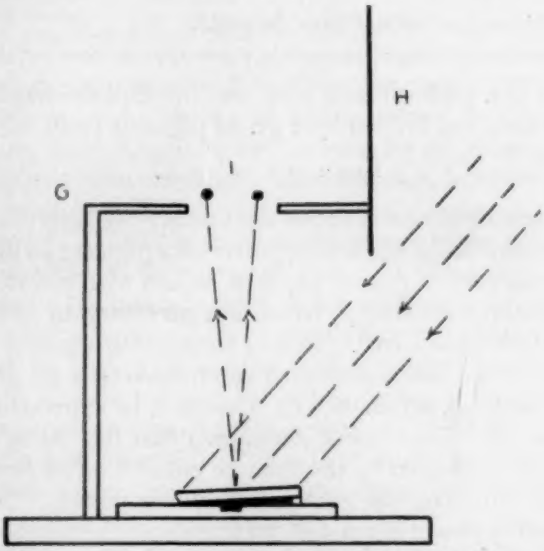
tively, to disappearance and appearance of the edge, yields the desired result. Since the fading away and reappearance is so gradual, due to the fact that the least perceptible increment of intensity which the human eye can detect is 1 to 2 per cent. (Fechner's Law), it is clear that no high degree of precision is attainable by this method. By taking ten pairs of readings it is found that the average deviation from the mean is about 3 to 4 per cent.

A suitable viewing device has been found necessary since the reflection of the observer's face in the plate *C* is annoying. An advantageous arrangement is shown in Fig. 2, where strong daylight illuminates the apparatus from the observer's right (or left). The essentials of the device are, a blackened board, *G*, about 30 cm. above the apparatus, with suitable openings for the eyes, *I*,

and a vertical blackened shield, *H*. The entire instrument is to be called "Cryptometer." [2] (*κρύπτω* conceal; *μέτρον* measure).

If it should ever become necessary to develop the instrument so as to yield results of high precision it will always be possible to do so. The two methods which suggest themselves are: (1) if we call the hiding complete when the brightness of the variable layer is 99 per cent. of that of the infinitely thick layer, then, by means of a photo-electric cell, which measures the light from the

FIG. 2.



two layers in contact at the edge *B* (Fig. 1), it will be possible to locate the desired point very accurately; (2) by making, preliminarily, a study of the variation of brightness of the paint with increasing thickness, a formula may be established connecting these two quantities; then, by finding a position of the wedge where the brightness is, say, one-half that of the infinitely thick layer, it is possible to calculate the thickness yielding 99 per cent. of the brightness of the infinitely thick layer. These methods have not been developed for the reason that the cryptometer in its present form is not only sufficiently accurate for most purposes but is extremely simple in its construction and operation.

[2] I am indebted to Mr. H. Green for suggesting this name.

A sharp distinction must be made between the hiding power of a pigment and that of a paint. Not only are these quantities expressed in different units, but they are not necessarily related in the sense that a pigment of great hiding power necessarily produces a paint of correspondingly great hiding power. Taking up first the hiding power of pigments, let us consider an intimate mixture of x grs. of a white pigment and y grs. of colorless (or very pale) linseed oil. This mixture is tested in the cryptometer and the critical thickness producing complete hiding is found. Let

t = thickness of critical layer (in cms.).

b = numbers of grs. of pigment in a disc of 1 cm.^{2} base and thickness t.

Then, if b grs. pigment hide 1 sq. cm., we find the number of sq. cm. A covered and hidden by 1 gr. of pigment from the relation

$$b : 1 :: 1 : A \text{ or } \frac{1}{b} = A.$$

Since the hiding power is better the thinner the layer, *i.e.*, smaller than b, we may define the hiding power of a pigment as the reciprocal of the number of grs. of pigment, mixed with colorless linseed oil to painting consistency, which are necessary to hide a black, non-absorbent area 1 cm^2. This is numerically equal to the number of square centimetres covered and hidden by 1 gr. of pigment. Hiding powers of pigments will, therefore, be expressed in terms of cm^2. per gr. Experiment has shown that the hiding power of the pigment is affected by the relative amount of oil present—the thinner the mixture, the greater the hiding power. This means that the hiding power is not only a function of the number of white particles, but also of their separation. In order to reduce all mixtures to a standard condition, the terms "painting consistency" are specified in the above definition. Data will be presented later.

Turning next to the hiding powers of paints, we choose, as a measure, the number of sq. ft. which one gallon of paint will cover and hide. While the retention of the metric system might, at first thought, seem advisable, practice dictates the selection of the English system of units. The actual measurements and calculations are very simple. The paint is measured up in the cryptometer and, from a knowledge of the critical thickness, the number of square feet per gallon is calculated at once. As a matter of fact, cryptometers are now made so that one may read the result directly off a separate scale etched on the lower plate.

The results obtained for characteristic pigments and paints are summarized in the following table:

Pigment.	grs. pigment / grs. oil.	Hiding-power pigment.	Hiding-power paint.
		cm² / gr.	sq. ft. / gal.
1. Sublimed white lead	7:3	26	172
2. Basic carbonate white lead	7:3	36	243
3. Lithopone	4:3	51	190
4. Titanox	3:4	58	132
5. Zinc oxide (leaded)	1:1	59	194
6. Zinc oxide (pure)	3:4	57	145
7. Zinc oxide plus trace of lamp black	3:4	190	486

Concerning the pigments themselves, the results obtained speak for themselves. The theory of hiding power as dependent on particle size, wave-length of light, character of vehicle, etc., will be taken up in a separate paper. The one point, however, to which attention must be called, is that involving a comparison of pigments 6 and 7. The only difference between these two pigments is that 6 is white while 7 is pearl gray. If these same pigments are compared by rubbing them down with oil and ultramarine blue, the gray sample will show the darker tint; hence it will be considered as having a smaller hiding power than the white sample. As a matter of fact, the hiding power of the gray sample is more than three times as great as that of the white. In a paper describing a new colorimeter,[3] it will be shown that all paints are more or less gray. It is, therefore, clear that, since the rub-down test yields results which are violently at variance with those obtained with the cryptometer, the former method must be discarded.

The results on paints are interesting. While the hiding powers of zinc pigments are larger than those of the white leads, the reverse is true of the paints. (Compare samples 2 and 6.) The reason is obvious when the "oil absorption" of the pigments is considered. One pound of zinc oxide mixed with one pound of linseed oil yields a mixture of proper painting consistency, while a similar mixture of white lead in oil is entirely too "runny" for painting purposes. More lead must be added to give the correct "body" or thickening to the paint. Such a paint has the greater

[3] Presented at the fall meeting of the National Academy of Sciences, Baltimore, 1918; also at meeting of the Optical Society, Baltimore, Dec. 28, 1918.

hiding power, but it has gained its superiority in consequence of the excess of lead pigment which has been added.

While ideal conditions exist in the cryptometer tests, it seemed of interest to compare the results thus found with those obtained in actual painting practice. I am indebted to Mr. R. J. Hauk for preparing the paints and carrying out the painting tests. Briefly, the experiment consisted in preparing three zinc paints (pigment ground in linseed oil). Three clapboard panels, having an area of 3.75 square feet, were well primed with white paint. A cross of black paint was applied to each of these surfaces and the whole was allowed to dry. Then successive coats of the respective paints were applied until the black cross could no longer be seen. From a knowledge of the area painted and the volume of paint consumed in each case it was possible to calculate the number of square feet per gallon which the respective paints would cover and hide. These tests extended over a week or more. When about half the adequate number of coats had been applied, the writer made hiding-power tests with the cryptometer. The results were as follows:

Paint.	No. of coats.	Hiding-power painting test.	Hiding-power cryptometer.
		sq. ft. / gal.	sq. ft. / gal.
1. ZnO (heavily leaded)..63.2 per cent. Vehicle.....36.8 per cent.	6	225	256
2. ZnO(less heavily leaded)52.7 per cent. Vehicle.....47.3 per cent.	7	204	213
3. ZnO (pure).....48.6 per cent. Vehicle.....51.4 per cent.	9	150	150

On the whole, the agreement is quite good. The findings of the painting test will always be the smaller for two reasons:

1. Brush marks will leave thin places which must be covered up.

2. Only whole numbers of coats may be applied, *i.e.*, if theoretically 6.4 coats would just hide the black cross it is unavoidable that 7 coats be applied. Obviously, the greater the number of coats, the closer is the approach to the ideal conditions which are realized in the cryptometer.

The conclusion to be drawn from these experiments is that the true hiding power of a paint may be obtained in a very few minutes by means of the cryptometer.

It is clear that if a surface of lighter tint (say, white pine) is to be hidden, the number of square feet per gallon will be considerably larger. The cryptometer must necessarily leave out of consideration the absorption of the paint in the pores of the wood. However, this instrument may be adapted to take care of lighter backgrounds by removing the black paint from the lower surface of plate (*A*), Fig. 1. If this plate be rested on the actual surface to be painted, it is again possible to determine in advance just how many gallons are required to paint a given area.

The principal purpose of this discussion is to present the cryptometer rather than final values of the hiding power of pigments and paints. In order to realize the latter, it will be necessary to grind the pigments in oil and to formulate a rigorous definition of, and test for, "painting consistency."

The results obtained may be summarized as follows:

1. An instrument which yields numerical values of the hiding powers of pigments and paints has been devised.

2. A table of true hiding powers of characteristic pigments and paints is presented.

3. A comparison of actual painting and cryptometer tests shows that the two are essentially in agreement.

(This work was carried out partly at the Johns Hopkins University and partly at the Research Laboratory of the New Jersey Zinc Co.)

September 30, 1919.

Helium Production in the United States.—Bulletin 178*c*, being part of the advance sheets of Bulletin 178, about to be issued by the Bureau of Mines, gives at some length the work that was done during the war in the search for sources and methods of extraction of helium. No more romantic story is likely to be developed in connection with the labors of the Chemical Warfare Service and War Industries Board than that set forth in this account. The value of helium is simply, that being uninflammable and next in lightness to hydrogen among known substances, it serves excellently for balloons.

Even in the discovery of helium there is a most interesting romance which serves to show the connection between the most important and even tragic phases of life and pure research. Helium was first recognized in the sun and for a long while no terrestrial occurrence was known. In 1868 Janssen observed a yellow line in the spectrum of the solar chromosphere, near, but not coincident with, the sodium lines. In 1889 Hillebrand noticed the evolution of inert gases from uraninite, which he supposed to

be an alltropic form of nitrogen. In fact, these gases do contain nitrogen, but it remained for Ramsay and Crookes, operating with cleveite, a variety of uraninite, to obtain a gas which gave the same line that Janssen had observed. Frankland and Lockyer, who had followed up Janssen's work, had given the name "helium" to the substance causing the line.

It was soon found that helium is widely distributed on the earth but in small amounts. It has been proved to be the end-product of the emanations of radio-active substances. In 1915 Ramsay wrote to Moore, of the Bureau of Mines, stating that he had been endeavoring to find sources of helium for use in airships, but had found none in England. He stated that he would obtain samples of mine and other natural gases from Canada and the United States. As the United States was then a neutral country, no special attention was paid to the subject, but Moore recalled that in 1907 Cady and McFarland had published a paper in the *Jour. Amer. Chem. Soc.*, showing the presence of helium in natural gas from Kansas. The subject became of great importance in 1917, when the United States entered the war, and many experts were consulted with a view to securing sources of helium and methods of obtaining it in quantity. These efforts are described in much detail in the bulletin, among which is the information that in the summer of 1917 Dr. R. B. Owens, Secretary of The Franklin Institute, then a captain in the Signal Service Corps, was sent abroad on a special mission, but being much interested in the helium problem he took with him a special letter to the British Admiralty, which resulted in the sending of two British naval officers to investigate the subject.

The ultimate result of the efforts of the many experts and civil and military officials was that large amounts of nearly pure helium were obtained from natural gas. At the time the armistice was signed, about 200,000 cubic feet of gas containing 92.5 per cent. of helium had been produced and stored in steel cylinders under 2000 pounds' pressure. Seven hundred and fifty of these cylinders, each containing enough gas to make 200 cubic feet at atmospheric pressure, were on the dock at New Orleans ready for shipment to France, when the cessation of hostilities occurred.

H. L.

The Electric Furnace Defended. H. G. WEIDENTHAL. (*American Electrochemical Society*, 1919.)—A polemic defending the electric furnace for steel-making against prejudices of the trade. The user is urged to acquaint himself thoroughly with electric furnace practice and furnish the best that can be produced by the electric furnace. Thus "he will do justice to himself and his business, and at the same time give the electric furnace a square deal."

DEVELOPMENT OF AN AIRCRAFT INCIDENCE METER.*

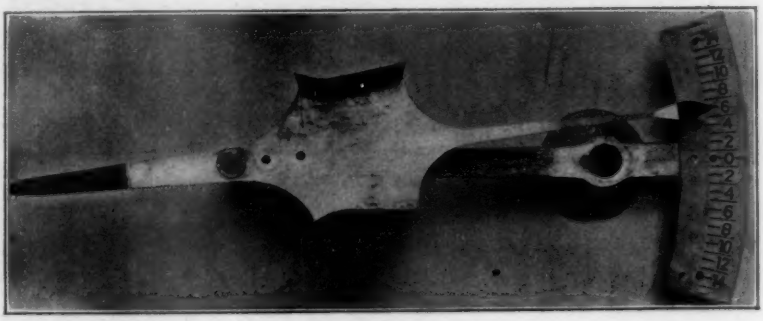
BY

A. F. ZAHM, Ph.D.

Navy Department, Bureau of Construction, and Repair.

Preface.—To enable the air pilot to read at a glance the direction of flow of the air past his airship or airplane, a balanced weathervane indicating promptly small changes of incidence has been developed and tried under regular working conditions. The scale drawings and test of the device herein described were made respectively by Mr. L. H. Crook and Mr. S. S. Rathbun, members of the aeronautics staff at the Washington Navy Yard.

FIG 1.



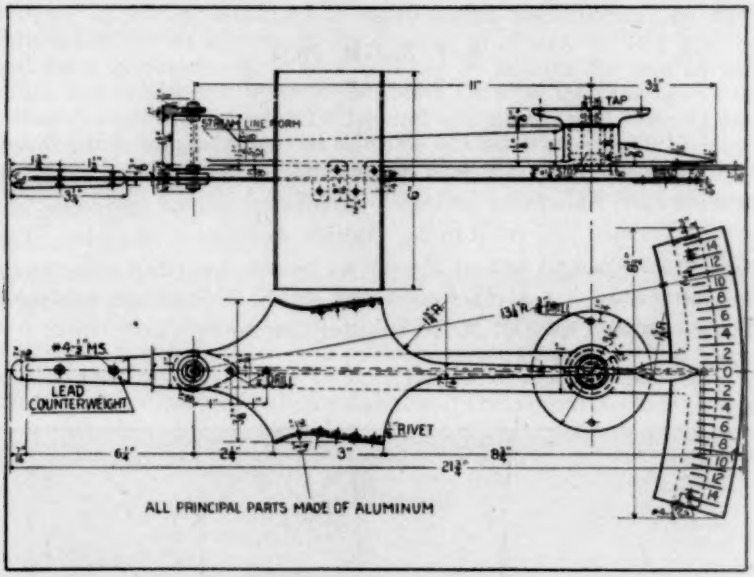
Aircraft incidence meter.

Model.—Figs. 1 and 2 give the general appearance and dimensions of this instrument. It consists of a two-blade weathervane supported on a horizontal pivot at the end of a bracket arm protruding forward from an airplane strut and adjustable in pitch by means of the clamping nut at its base. The vane has a forward counter-weight to insure static balance, and a pointer playing on a graduated arc of fourteen inches radius, indicating even degrees

* Communicated by the Author.

and readable to fractions of a degree from the pilot's seat. The blades have the sectional shape of an Eiffel Wing No. 5, which at zero incidence possesses very slight drag and a large increase of lift with slight increase of incidence.

FIG. 2.



Plan for aircraft incidence meter.

Wind Tunnel Test.—When the instrument was given its preliminary test in the $8' \times 8'$ tunnel its pointer remained steadily fixed in the wind direction until forcibly displaced. It then promptly returned to zero incidence without lag or indication of friction.

Observations with C. & R. Incidence Meter in Flight.

Speed of Flight in Knots	55	65	75
Normal variation.....	± 0.6°	± 0.4°	± 0.4°
Precision of reading possible.....	± 0.3°	± 0.2°	± 0.2°
Occasional variation..	± 2.°	± 1.°	± 1.°

Test in Flight.—The instrument was finally mounted midway between planes on the nearest right-hand strut of Flying Boat HS-2 No. 1840, and carried through very still air at three different fixed speeds. The preceding table indicates its behavior under these circumstances.

Conclusion.—If this instrument is to be put into use, it may be lightened somewhat and provided with a strap to lash its flange to the aeroplane strut. So finished, it would weigh about 1.5 pounds.

The World's Merchant Tonnage.—The weekly edition of the *Berliner Tageblatt*, of September 10 last, gives some statistics from the recently published Lloyd's "Register of Shipping," this being the first issue of that maritime text-book published without censorship since the outbreak of the war. The data show that the total merchant tonnage is slightly higher than in 1914, in spite of the great destruction that has been going on, but the distribution under the several flags has been materially changed. In July, 1914, the British flag covered 41.6 per cent. of the total merchant tonnage, the German, 11.1 per cent., and the United States, 9.4 per cent. The American shipping was probably made up largely of the coastwise vessels of the Great Lakes and Atlantic seaboard, with those used in the trade with the West Indies and South America, for the high-seas fleets (the great liners) were mostly under British and German control. In July, 1919, the figures are: British, 34 per cent.; United States, 24 per cent. Germany, of course, has passed into a minor position. The German commentator derives some satisfaction in the case by the view that Great Britain in crushing her closest rival, in pursuance of the "rule the waves" policy, has reared a more dangerous adversary, namely, the United States.

Attention is, however, called by the commentator to the fact that many of the ships now afloat are "old tubs," that would have long since gone to the scrap heap if the scarcity of tonnage had not made use necessary. It is probable, therefore, that in a year or two considerable amortization of tonnage will occur, the replacement coming largely from American and Japanese yards.

H. L.

Cranes. CHESTER C. RAUSCH. (*National Safety Council*, 1919.)—Ever since Noah handled lumber while constructing the Ark and the Egyptians stones while building the Pyramids, the problem of handling material has been present in industrial work of whatever sort or kind. It is not improbable that one of the earliest methods of handling material was by the use of some form of rope manufactured of hide or woven barks passed over

the limb of a tree. Later some means was found to use parts of trees that had been blown over, or that had in some way been felled, to construct the simpler forms of derricks. As we know them to-day derricks and cranes are the result of a development extending through centuries and representing the solution of many problems one after another. Of late years various forms of power have placed at the disposal of man a ready means for adapting his machines to types of work that were formerly done by the laborious efforts of man and beast.

In no particular field has greater development been made than in that of handling and conveying materials in connection with the process of construction, manufacture, and transportation. More recently the fact that electric current furnishes such a flexible source of power has greatly accelerated this development and in the past decade the enormous increase in the use of steel and iron for the fabrication of structures and objects formerly made of wood has called for a further development of machinery to handle the greater unit weights involved. During the last three or four years particularly the production of cranes of all sorts has increased unbelievably and not only is their lifting capacity greater but their flexibility of operation and their safety of manipulation as well. Where we formerly saw a braced pole derrick superseding a single stick or gin-pole, we now find locomotive cranes, tower derricks, overhead travelling cranes, gantry cranes, floating derricks, wall cranes, gib cranes, and a variety of combinations of these devices that enable almost any conditions where material must be handled to be met by the selection of a type of crane to do the work and that will, at the same time, consume the least power, offer the greatest flexibility, and afford the greatest safety during operation.

Motor Transport Training Schools.—The United States Army is definitely launched in the field of vocational training for the motor transport corps. Men skilled in automobile vehicle operation and repair do not exist in anything like adequate numbers for the requirements even of civil life; and the war with Germany has demonstrated that no matter how good our Army may be in other respects, its efficiency will be conditioned by that of the motor transport branch.

Therefore, the Army is organizing schools to train men in the various branches of automobile repair, construction, and operation. There are schools under trained teachers where the time of the pupil is wholly devoted to receiving instruction.

Apart from the military necessity, the automobile industries will benefit by the establishment of this training system.

The United States Civil Service Commission is receiving applications to fill 150 positions of assistant instructors in motor transport training schools.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

AN APPARATUS FOR MEASURING THE RELATIVE WEAR OF SOLE LEATHERS AND THE RESULTS OBTAINED WITH LEATHER FROM DIFFERENT PARTS OF A HIDE.[1]

By R. W. Hart and R. C. Bowker.

[ABSTRACT.]

A SHORT paper dealing with the development of a simple machine for testing the wear resistance of sole leathers and giving the results of extensive research work on the relative resistance to wear in different parts of a bend. The machine, which is described at various stages of its development up to the present time, was designed to subject leather to an abrasive action similar to that which it undergoes on a shoe sole, and to accelerate the wear so that a test could be completed in 24 hours.

The result of three extensive tests on the resistance to wear in different parts of a bend are given, with detailed description of the manner in which these tests were made, one test showing the results of comparative wearing tests both on the machine and in actual service are included in this paper. The results of all investigations show that the laboratory testing machine gives a proper indication of the wearing resistance of leathers.

EQUILIBRIUM CONDITIONS IN THE SYSTEM CARBON, IRON OXIDE, AND HYDROGEN, IN RELATION TO THE LEDEBUR METHOD FOR DETERMINING OXYGEN IN STEEL.[2]

By J. R. Cain and Leon Adler.

[ABSTRACT.]

IT is shown that mixtures of iron oxide and Acheson graphite are not, and mixtures of iron oxide with "cemented" iron or white iron (annealed or unannealed) are, reduced at 900° C. by

* Communicated by the Director.

[1] Technologic Paper 147.

[2] Scientific Paper 350.

the carbon in them when hydrogen is passed over them at rates of two litres per hour or faster. Because of these facts, it is probably impossible to determine by the Ledebur method more than 75 per cent. of the oxygen present in steels as ferrous oxide. The effect of rate of passage of hydrogen on the Ledebur oxygen content of certain steels is shown.

THE COKING OF ILLINOIS COAL IN KOPPERS TYPE OVENS.[3]

By R. S. McBride and W. A. Selvig.

[ABSTRACT.]

(An Operating Test at the St. Paul Plant of Minnesota By-Product Coke Co. Joint Report of National Bureau of Standards and Bureau of Mines.)

THE great importance during the war period of substituting Mid-continent coal for coals from more distant sources, even in by-product coke oven work, was well recognized. The Bureau of Standards was ordered to conduct an investigation of a new coke oven process claimed to be especially suited to this purpose and in connection with this the Bureau was requested to conduct a test of the St. Paul plant of the Minnesota By-Product Coke Company which is owned by The Koppers Company, Pittsburgh. The Bureau of Standards, in coöperation with the Bureau of Mines, carried out this operating test, using about 7600 tons of coal from the Orient Mine, Franklin County, Illinois. All phases of coal handling, by-product recovery and laboratory tests were under observation by the staff of 37 Government engineers and chemists employed on the work. In addition, those in charge had the benefit of advice and comment from a considerable number of experts who are specialists in the field of coke oven operation.

The quantity of all coal used and of all by-products obtained was carefully weighed or measured at regular intervals and samples of each material were taken for analysis. The Bureau of Standards was responsible for the general planning and supervision of the test work. Its representatives made all observations of battery operation, high temperature measurements, by-product recovery, and chemical laboratory work on gas and by-products.

[3] Technologic Paper No. 137.

The Bureau of Mines was responsible for the sampling of the coal both as it was loaded at the mine and as dumped at the plant. It supervised the weighing, coal handling, coke handling, and coke sampling operations, and made all analyses of coal and coke. Its representatives also made general observations on the character of the coke and operation of the ovens.

The Minnesota By-Product Coke Company Plant consisted of 65 ovens of 18¼ inches average width, operating during the test period with an average gross coking time of 19 hours and 33 minutes with coal finely pulverized, 12.75 tons per oven as charged. The temperature of the heating wall as determined by rare metal thermocouples averaged about 1200° during the test period. The coke was screened to produce large and small furnace sizes, stove, nut and pea domestic sizes, and breeze. The gas was separated into rich and lean at the battery. Separate test records were kept of each size of coke and of each quality of gas. Practically all of the ammonia produced was made up into sulphate immediately through the direct recovery process. Although the plant operated for the production of pure light oil products only the total production of light oil was measured, but the yield of various constituents was determined by analysis.

The coke produced was very irregular in size, had a longitudinal fracture, was fingery, brittle and shattered easily. The cell structure was very small and regular. The coke was lighter than the average by-product coke, weighing only 23 pounds per cubic foot. The first table summarizes the characteristics and yield of this coke further. The large percentage of domestic sizes obtained and therefore the unusually small percentage of furnace size demonstrates that the coke will not stand handling and screening without breaking up into many smaller sizes. This is due primarily to its fingery and brittle characteristics.

In order to test the behavior of the furnace-size coke produced from the Orient coal, arrangements were made to use about 1800 tons of this material in the blast furnace plant of the Mississippi Valley Iron Company. The coke regularly used at that furnace is produced in Koppers ovens of the Laclede Gas Light Company, St. Louis, from a mixture of Elkhorn, Pocahontas (low volatile) and Illinois coals. The substitution of the Illinois coke for the regular supply was accomplished abruptly and continued without interruption throughout the ten-day period. Several experts were

present during this test, and it was the unanimous opinion of these persons and of the blast furnace operators that the Illinois coke had shown highly satisfactory results. However, it should be borne in mind that the furnace used for this test was of small capacity, and it is not certain, therefore, that the results in this case would be duplicated on a large-size furnace.

The yields of gas and by-products are summarized in the second table.

As a result of the test it is clearly demonstrated that some of the Illinois coals can be coked in the Koppers-type oven without radical change in operating methods for the production of coke which can be successfully used in a blast furnace. However, it appears that the temperature at which Illinois coal should be handled for the production of the best coke is somewhat lower than the best operating temperatures for Eastern coals, and moreover, the speed of coking of the Illinois coal is somewhat less. The yield of gas and by-products from Illinois coal of the kind tested is excellent both in quantity and quality. Of course the coal tested in this case represents one of the best Illinois coals for coking purposes, being lower in ash and sulphur and otherwise superior to many from this field.

In general the comparison of Eastern coking coals with those from the Mid-continent field must be made upon an economic basis since which source will be preferable depends altogether on local conditions which will affect the cost of the material and the relative expense of handling. These phases of the question have, however, not been discussed in this report.

GAS AND BY-PRODUCT YIELD SUMMARY.

Product	Per ton of coal	
	As charged	Dry
Gas:		
Surplus .....cubic feet	5490	5970
To ovens .....cubic feet	5230	5690
Total .....cubic feet	10720	11660
Tar:		
As produced .....gallons	7.81	8.49
Dry (computed) .....gallons	7.57	8.23
Ammonium sulphate—pure.....pounds	27.88	30.33
Ammonia .....pounds	7.19	7.82

Light oil: Product	Per ton of coal: As charged	Dry
As produced ..... gallons	3.71	4.03
Under 200° C. ..... gallons	3.11	3.38
Benzene ..... gallons	2.105	2.289
Toluene ..... gallons	0.497	0.540
Solvent Naphtha ..... gallons	0.130	0.141

COAL AND COKE SUMMARY.

Coal used

As charged	7685.6 tons
Dry	7065.4 tons
Oven charged	603

Coal per oven

As charged	12.75 tons
Dry	11.72 tons

Coking time

Average gross	19 hr. 33 min.
Average net	19 hr. 11 min.

Coke produced (dry)

Furnace	2704.4 tons
Stove	1178.1 tons
Nut	549.5 tons
Pea	89.9 tons
Breeze	308.4 tons
Total	4830.3 tons

Ratio of dry coke to dry coal

Furnace	38.3 per cent.
Stove	16.7 per cent.
Nut	7.7 per cent.
Pea	1.3 per cent.
Breeze	4.4 per cent.
Total	68.4 per cent.

SIZES OF COKE PRODUCED.

Furnace	56.0 per cent.
Stove	24.4 per cent.
Nut	11.4 per cent.
Pea	1.8 per cent.
Breeze	6.4 per cent.
Total	100.0 per cent.

SOME TESTS OF LIGHT ALUMINUM CASTING ALLOYS; THE EFFECT OF HEAT TREATMENT.[4]

By P. D. Merica and C. P. Karr.

[ABSTRACT.]

THE tensile properties and the hardness of a number of different compositions of light aluminum casting alloys have been determined, the resistance to corrosion compared and the resistance to the action of alternation or vibratory stresses determined on a few commonly used compositions.

It is advisable to use for tensile tests a test bar cast almost to size; a bar cast with the test length $^{9}/_{16}$ inch in diameter, after-

[4] Technologic Paper No. 139.

wards machined to .505 inch gives satisfactory results. The use of a type of test specimen which can be gripped in a self-centring holder in the testing machine is recommended.

A study of the effect of chemical composition on the mechanical properties has shown that it is possible to obtain an alloy containing from 2 to 3 per cent. of copper together with 1 or 2 per cent. of nickel, manganese or both, which will have a reasonable amount of ductility, and it is believed that an alloy of this type should have commercial value. Tensile properties suggested for such an alloy are the following:

Tensile strength.....20,000 to 25,000 pounds per square inch
Elongation in 2 inch.....not less than 5 per cent.

The addition of magnesium to alloys containing copper reduces in a marked manner the ductility, but increases the tensile strength and the hardness.

The effect of heat-treatment on test bar castings, consisting of annealing 500° C., cooling them in air from this temperature and allowing them to age for several days, is to increase the tensile strength and the hardness; the ductility of the alloy is generally decreased, but may in some cases be increased. The presence of magnesium in the alloy in amounts of from 0.5 to 1.5 per cent. seems to increase the hardening effect of heat-treatment. The heat-treatment of light aluminum castings would seem to have commercial possibilities.

The microstructure of the different alloys was studied and it was found that fracture in them prefers a path along the brittle envelopes surrounding the grains of aluminum, consisting of the various eutectics which are formed with the added metals or their compounds.

Two months' exposure in the salt spray produced only slight corrosion of several compositions of cast alloys. There was no appreciable difference between the different compositions in resistance to corrosion.

A study of the resistance to the action of alternating stresses of three compositions of light cast alloys:

E series—containing 8 per cent. copper
Z series—containing 2-3 per cent. copper, 12-15 per cent. zinc
G series—containing 1.5-2 per cent. copper, 1.5-2 per cent. manganese,

showed that there was no marked difference in the behavior of the three alloys in the test although the E series was somewhat superior and the G series somewhat inferior to the others. All of the three alloys will withstand 10,000,000 complete reversals (tension to compression) at a maximum fibre stress of 7000 pounds per square inch.

AN ELECTROLYTIC RESISTANCE METHOD FOR DETERMINING CARBON IN STEEL.[5]

By J. R. Cain and L. C. Maxwell.

[ABSTRACT.]

METHOD and apparatus are described for rapidly and accurately determining carbon in steel by absorbing in a solution of barium hydroxide the carbon dioxide resulting from direct combustion of the metal in oxygen, and deducing the carbon content from the change in electrical resistance of the barium hydroxide solution.

THE DIRECT DETERMINATION OF INDIA RUBBER BY THE NITROSITE METHOD.[6]

By John B. Tuttle and Louis Yurow.

[ABSTRACT.]

THE method of determination of india rubber described by Wesson (Tech. paper 35) was applicable only to compounds containing new rubber, and even for these was not always satisfactory. By varying the procedure, it has been found possible to extend the usefulness of the method to compounds containing reclaimed rubber and substitutes, as well as lamp-black and bituminous substances. The method is based upon the formation of rubber nitrosite, which is purified, burned in a combustion furnace and the carbon dioxide formed is calculated to rubber.

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[5] Technologic Paper No. 141.
[6] Technologic Paper No. 145.

Magnetic State of Prehistoric Burned Earths. P. L. MERCANTON. (*Bull. Société Vandoise*, vol. lii, No. 194.)—The author is continuing the investigations of the Italian, Folgheraiter. When pottery was baked or lavas cooled they assumed in many cases a magnetic state depending upon the direction of the earth's magnetic field at the time. If the position which they occupied while cooling can be determined, it is sometimes possible to infer the direction of the earth's magnetic field after a magnetometric investigation of the object. M. Mercanton reports in this paper the results of his examination of eleven pieces of baked clay, used as sinkers in fishing. He finds no regularity of magnetization and concludes that all were baked at a time when the earth's field was horizontal, *i.e.*, when the inclination was zero. He is surprised at this, since the sinkers come from various ages, such as the stone, and the bronze age.

G. F. S.

Signal Corps School.—Realizing that there are comparatively few trained radio operators available, to fill the needs of the Signal Corps, the Government has established a training school at Camp Alfred Vail, New Jersey. Here men with only common school education can be sent from the enlisted personnel and secure a thorough training in signal corps specialties. These in the main are the same as one meets in civil life, with the addition of their military application. Men will be trained as telegraph, telephone and radio operators; electricians, instrument makers and repairers; cable splicers, cooks, clerks, photographers; chauffeurs and motorcyclists.

The location of this school at Camp Vail, N. J., will be considered as temporary until the extent of the needs of the Army with reference to permanent Signal Corps Schools has been definitely determined and approved by the War Department.

Passenger Train Resistance. (*University of Illinois, Engineering Experiment Station, Bulletin 110*, 1919.)—Tests to determine the resistance of passenger trains at all speeds up to seventy miles per hour, and for average car weights have been completed by the Railway Department of the Engineering Experiment Station of the University of Illinois. Of the 240 cars composing the 28 trains tested, 178 had six-wheel trucks, and 62 had four-wheel trucks. These tests were made on the lines of Illinois Central Railroad by Professor E. C. Schmidt and H. H. Dunn upon well constructed and well maintained main line track laid almost entirely with 85-pound or 90-pound rail and ballasted with broken stone.

From the results, which are given in Bulletin No. 110 of the Engineering Experiment Station, a table has been prepared showing the probable average value of resistance for passenger trains composed of cars weighing from thirty to seventy tons and operating at speeds ranging from five to seventy-five miles per hour.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

BOTULISM FROM CANNED ASPARAGUS.[1]

By Charles Thom, Ruth B. Edmondson, and L. T. Giltner.

[ABSTRACT]

CANNED asparagus taken from the jar and served as a salad caused the deaths of four persons in Boise, Idaho, in January, 1919. Cultures from a portion of the salad and from one jar of asparagus canned in the same pack as that used in the salad showed a strain of *B. botulinus*, designated for convenience the "Boise strain." Animal experimentation proved that this strain produces a very virulent toxin. Guinea-pigs, rabbits, and chickens are readily susceptible to the toxin when taken by mouth. Cattle, pigs and dogs, while not susceptible to feeding, succumb to injections of the toxin. The strength of the toxin produced by this strain varies with changes in cultural conditions. At the highest toxicity obtained the minimum lethal dose for a guinea-pig weighing from 330 to 400 grams was an intraperitoneal injection of 0.0001 c.c. of a filtered dextrose beef infusion culture incubated 28 days at 35° C.

The toxin is destroyed by heating to 75° C. or by heating at 73° C. for 10 minutes. Bacilli freed from toxin by washing or spores freed from toxin by heat do not produce symptoms of poisoning when fed or injected, but have been recovered from the fæces of fed animals in virulent form. All cultures of this strain have a characteristic offensive odor (putrefactive), which was clearly evident in the canned material connected with this case.

The organism is a heat resistant anærobe, whose optimum temperature is 37° C. It will live and multiply, however, at 12° C. (ice-box temperature). Its spores survive ordinary boiling for an hour or boiling at 10 pounds pressure for 15 minutes. Such an organism, if present, may be expected to survive the heating used in canning and to produce marked evidences of spoilage in the can. When physical evidences of such spoilage are present, the material should be destroyed.

* Communicated by the Chief of the Bureau.

[1] Published in *J. Amer. Med. Ass'n.*

Dust Hazard in the Abrasive Industry.—The conviction is growing among experts, that workers in dusty trades are especially liable to tuberculosis. The subject has been treated in a bulletin by Professor Winslow, of the Yale Medical School, assisted by L. and D. Greenburg of the U. S. Public Health Service and published as Reprint 530 of that service. Extensive examinations were made of two large factories designated respectively as B and C, the former employing 2000 hands, of which 400 are women. In both cases much defect was found in the methods of avoiding dust. While ordinary inspection in one of the factories in which some installations had been made with a view of improving the conditions some benefit appeared, the analysis of the air did not confirm the opinion. Comparisons are given between these factories and those of other occupations in which dust is produced in considerable amount. From the figures it seems that the amount of dust ranges from 2.37 mgms. of solid particles in 100 cubic feet of the air of a good polishing shop, to 780.5 mgms. in the same volume of air in factory C. It is noted that while the total dust in the air of a carpet and blanket mill is higher than most of the other industries given in the table (such as pottery, asbestos, tobacco, steel grinding), the inorganic matter in the textile dust is the lowest. The bulletin concludes with the statement that establishments in which abrasive materials are made may present conditions in regard to aerial dust unequalled in any other industry, and that the subject is an important one in industrial hygiene. It would be worth while to inquire whether any form of gas-mask can be utilized for the protection of the worker.

H. L.

Heartwood and Sapwood. (*Technical Notes, Forest Products Laboratory,* 1919.)—In over 300,000 tests which have been made at the Forest Products Laboratory, Madison, Wis., on the various species of wood grown in the United States, no effect upon the mechanical properties of wood due to its change from sapwood into heartwood has ever been noticed. Any difference in the strength of heartwood and sapwood can usually be explained by the growth and density of the wood.

In other than mechanical properties, there are differences between heartwood and sapwood which have an important bearing on their use for various purposes. The sapwood of most American species is considerably less resistant to decay than the heartwood, and where the wood is used without preservative treatment in situations which favor decay, the sapwood is likely to have a much shorter life. In these particular cases, therefore, strength requirements may have an indirect bearing on the choice between heartwood and sapwood, inasmuch as wood infected with decay is likely to have its strength properties, particularly that of shock resistance, greatly reduced.

THE FRANKLIN INSTITUTE.

(Proceedings of the Stated Meeting held Wednesday, October 15, 1919.)

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, October 15, 1919.

PRESIDENT DR. WALTON CLARK *in the Chair.*

Additions to the membership since last report, 16.

Reports of progress were presented by the Committee on Library, and the Committee on Science and the Arts. The President then announced that the Philadelphia Section of the American Institute of Electrical Engineers was meeting jointly with the Institute, and requested Prof. C. E. Clewell, Chairman, to preside.

Professor Clewell then introduced Captain S. W. Bryant, U. S. N., Acting Director Naval Communication Service, Navy Department, Washington, D. C., who presented a communication entitled "The U. S. Naval Communication Service." The lecture included a description of the formation of the Naval Communication Service and a general outline of its functions; status of this service at the time of the declaration of war and the demands made upon it during the war. Consideration was given to the methods of communication between Naval Headquarters and the fleet and between the fleet and its various units. A description was also given of the high-powered stations and their use during the war, and the development of radio direction finding stations was outlined with an indication of their use in war. Various other activities of the Naval Communication Service were described. The subject was illustrated by lantern slides. After a brief discussion the thanks of the meeting were conveyed to the speaker.

Adjourned.

R. B. OWENS,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of the Stated Meeting held Wednesday, October 1, 1919.

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, October 1, 1919.

MR. BENJAMIN FRANKLIN *in the Chair.*

The following reports were presented for final action:

No. 2728: Landreth Electrolytic Sewage Process. Protests considered. Howard N. Potts Gold Medal awarded to Clarence P. Landreth of Philadelphia, Pa.

No. 2714: Snook X-ray System. Edward Longstreth Medal of Merit awarded to H. Clyde Snook of New York City.

No. 2729: Simplex Fluid Meter. Edward Longstreth Medal of Merit awarded to John Walter Ledoux of Philadelphia, Pa.

The following report was presented for the first reading:

No. 2743: Raymond Concrete Pile.

R. B. OWENS,
Secretary.

SECTIONS.

Mechanical and Engineering Section.—A joint meeting with the Aëro Club of Pennsylvania was held on Thursday evening, October 2, 1919, Mr. Charles Day and Mr. Joseph A. Steinmetz presiding jointly. Brig. Gen. William Mitchell, Division of Military Aëronautics, War Department, Washington, D. C., presented a communication entitled "The American Air Service at the Front." The speaker described the development of aviation at the beginning of the European War, its progress up to the time the United States entered it, the necessity for basing technical and production development of aircraft and accessories on tactical requirements, conditions that existed from a technical standpoint before the entrance of the United States into the war, the formation of the American Air Service in Europe, its employment, use, tactics it developed and its work in combination with other Air Services. Consideration was also given to the future of the Air Service in this country. The subject was illustrated by numerous lantern slides. The thanks of the meeting were conveyed to the speaker.

Adjourned.

R. B. OWENS,
Secretary.

Section of Photography and Microscopy.—A joint meeting of the section with the Aëro Club of Pennsylvania was held on October 9, 1919, at 8 o'clock. Dr. Harry F. Keller and Mr. Joseph A. Steinmetz presiding jointly. The paper of the evening, entitled "Photography from the Airplane," was presented by Major Herbert E. Ives, Ph.D., of the Aviation Section, Signal Reserve Corps, lately in charge of Photographic Experimental Work, United States Air Service. The speaker gave an account of the beginning of Photography from the airplane, which occurred at the time of the beginning of the great war, and early became one of the most important activities of all the air forces. By its aid complete detailed maps of enemy trenches, batteries, and lines of communication were always available, thus entirely destroying the factor of secrecy, and so revolutionizing military strategy. Numerous technical problems arose in connection with the methods, apparatus and materials to be used in this newest application of photography, and ultimately nearly every branch of scientific photography lent its aid. The speaker described the research work done for the United States Air Service, and illustrated the subject by

lantern slides of the apparatus used and also exhibited photographs taken over the battle lines. After a vote of thanks to Major Ives the meeting adjourned.

Adjourned.

R. B. OWENS,
Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, October 8, 1919.)

RESIDENT.

MR. G. BERKELEY REED, Inventor, The Gladstone, Philadelphia, Pennsylvania.

NON-RESIDENT.

DR. WILBUR M. STINE, Author, 334 West Main Street, Mechanicsburg, Pennsylvania.

CHANGES OF ADDRESS.

MR. J. C. BANNISTER, Lock Box 26, Boston, Massachusetts.
MR. CARL G. BARTH, 416 Parkside Avenue, Buffalo, New York.
DR. THOMAS D. COPE, Randal Morgan Laboratory of Physics, University of Pennsylvania, Philadelphia, Pennsylvania.
MR. S. B. ECKERT, Finance Building, 1428 South Penn Square, Philadelphia, Pennsylvania.
MR. HALCOLM ELLIS, 6 Rue de Hanover, Paris, France.
MR. E. L. GIBBS, 2004 Herschell Street, Jacksonville, Florida.
MR. GEORGE R. HENDERSON, 8 West 40th Street, New York City, New York.
MR. JOHN W. HILL, 3536 Mitchell Building, Cincinnati, Ohio.
MR. ANTHONY DEH. HOADLEY, Union College, Schenectady, New York.
MR. P. JUNKERSFELD, Stone and Webster Engineering Corporation, 147 Milk Street, Boston, Massachusetts.
MR. H. J. KREBS, P. O. Box 951, Wilmington, Del.
MR. WM. RICHARD LITTLETON, Merchantville, New Jersey.
MR. JOHN F. MCCOY, 3015 Midvale Avenue, Station Z, Philadelphia, Pennsylvania.
MR. JAVIER RESINES, Racquet Club, Philadelphia, Pennsylvania.
MR. FREDERICK W. SALMON, 1129 North 28th Street, Philadelphia, Pennsylvania.
MR. A. B. STITZER, in care of Republic Engineers, Inc., 60 Broadway, New York City, New York.
MR. C. C. TUTWILER, West Conshohocken, Pennsylvania.
MR. J. R. WATKINS, 1920 Vilas Street, Madison, Wisconsin.
MR. ROY V. WRIGHT, 398 North Walnut Street, East Orange, New Jersey.

NECROLOGY.

Mr. Charles Norris, Aldine Hotel, Philadelphia, Pennsylvania.
Mr. M. C. Smyth, 1700 Morris Building, Philadelphia, Pennsylvania.

LIBRARY NOTES.

PURCHASES.

BARTLETT, F. W., and JOHNSON, T. W.—Engineering Descriptive Geometry and Drawing. 1919.

FULLER, C. E., and JOHNSTON, W. A.—Applied Mechanics, 2 vols. 1915, 1919.

HELDT, P. M.—The Gasoline Automobile; Its Design and Construction. 3 vols. 1919.

PAGÉ, V. W.—Gasoline and Kerosene Carburetors, Construction—Installation—Adjustment. 1919.

GIFTS.

Advance Tool Company, Catalogue B. Cincinnati, Ohio, no date. (From the Company.)

Alabama Geological Survey, Bulletin No. 20. University, Alabama, 1918. (From the State Geologist.)

American Cement Tile Manufacturing Company, Booklet of Bonanza Cement Tile Roofing. Pittsburgh, Pennsylvania, 1917. (From the Company.)

Association of Manufacturers of Chilled Car Wheels, Catalogue. Chicago, Illinois, no date. (From the Association.)

Beaumont, R. H., Company, Catalogues Nos. 33, 34, 36, 38 and 41. Philadelphia, Pennsylvania, 1919. (From the Company.)

Bedford Foundry and Machine Company, Catalogue of Cranes and Derricks, no date. (From the Company.)

Benson Electric Company, Catalogue. Superior, Wisconsin, no date. (From the Company.)

Black and Decker Manufacturing Company, Catalogue, 1919. Baltimore, Maryland, 1919. (From the Company.)

Blair Engineering Company, Catalogue of Ports, Valves and Slag Pockets. Chicago, Illinois, no date. (From the Company.)

Blaw-Knox Company, Bulletin No. 200 of Blaw Cableways. Pittsburgh, Pennsylvania, 1919. (From the Company.)

Bradley and Turton, Ltd., Catalogue of Testing Machines. Kidderminster, England, no date. (From the Company.)

Brown, Bayleys Steel Works, Ltd., Catalogue of Die Blocks. Sheffield, England, no date. (From the Works.)

Buckeye Blower Company, Bulletin No. 101. Columbus, Ohio, no date. (From the Company.)

Buffalo Hoist and Derrick Company, Crane Catalogue No. 2 C, and Engine Catalogue No. 21. Buffalo, no date. (From the Company.)

Butterfield and Company, Inc., Catalogue No. 17. Derby Line, Vermont, 1919. (From the Company.)

Canada Department of Mines, Memoir 109. Ottawa, 1919. (From the Department.)

Carnegie Endowment for International Peace, Year-Book, 1919. Washington, District of Columbia, no date. (From the Endowment.)

Central Scientific Company, Catalogue C. Chicago, Illinois, 1919. (From the Company.)

Clapp-Eastham Company, Bulletin Y. Cambridge, Massachusetts, 1919. (From the Company.)

Cleveland Gas Burner and Appliance Company, Booklet of Barber Patented Gas Burners for Artificial and Natural Gas. Cleveland, Ohio, no date. (From the Company.)

Cleveland Milling Machine Company, Catalogue B of Cutters. Cleveland, Ohio, no date. (From the Company.)

Colorado Iron Works Company, Catalogue of Continuous Filtration with the Portland Filter. Denver, Colorado, 1918. (From the Company.)

Cooper Hewitt Electric Company, Bulletin No. 78-B. Hoboken, New Jersey, no date. (From the Company.)

Combustion Engineering Corporation, Bulletin G-1. New York, New York, 1919. (From the Corporation.)

Cornell University, Register for 1918–1919. Ithaca, New York, 1919. (From the University.)

Dayton Electrical Manufacturing Company, Bulletin No. 55. Dayton, Ohio, 1919. (From the Company.)

Dominion of New Zealand, Statistics for the Year 1917, Vol IV. Wellington, 1918. (From the Government Statistician.)

Dorr Company, Bulletin No. 13 of the Dorrco Pump. New York, New York, 1919. (From the Company.)

Earle Gear and Machine Company, Bulletins 71–73, inclusive. Philadelphia, Pennsylvania, no date. (From the Company.)

Electrical Engineers Equipment Company, Bulletin No. 102-A. Chicago, Illinois, 1919. (From the Company.)

Foxboro Company, Inc., Catalogue of Foxboro Thermometers. Foxboro, Massachusetts, 1919. (From the Company.)

Fuller Engineering Company, Bulletin No. 201. Allentown, Pennsylvania, 1919. (From the Company.)

Gifford-Wood Company, Bulletin No. 42. Hudson, New York, 1919. (From the Company.)

Gilbert and Barker Manufacturing Company, Catalogue of Oil Storage Systems. Springfield, Massachusetts, no date. (From the Company.)

Goodrich, B. F., Co., Catalogue of Rubber Hose and Tubing. Akron, Ohio, 1917. (From the Company.)

Grant Manufacturing and Machine Co., Booklet, Perfect Riveting. Bridgeport, Connecticut, 1917. (From the Company.)

Harrison Steel Castings Company, Catalogue of Open Hearth Steel Castings. Attica, Indiana, no date. (From the Company.)

Holcroft and Company, Catalogue describing Ovens and Furnaces. Philadelphia, Pennsylvania, 1919. (From the Company.)

Holden and White, Inc., Catalogue No. 20. Chicago, Illinois, 1919. (From the Company.)

Holz and Company, Inc., Bulletin No. 4. New York, New York, no date. (From the Company.)

Horton, John T., Company, Inc., Catalogue of Winches, Derricks and Cableways. New York, New York, 1919. (From the Company.)

Hydraulic Press Manufacturing Co., Catalogues Nos. 43 and 80. Mount Gilead, Ohio, 1919. (From the Company.)

India Geological Survey, A Bibliography of India Geology and Physical Geography, Parts 1 and 2. Calcutta, 1917–1918. (From the Geological Survey.)

Indian Railway Conference Association, Locomotive and Carriage Superintendents Committee, Proceedings for 1918. Calcutta, India, 1919. (From the Committee.)

Institute of Metals, Journal No. 1, 1919. London, England, 1919. (From the Institute.)

Instituto y Observatorio de Marina, Anales Meteorologicas, Magneticas y Sismicas, Seccion 2. Ano. 1914–1915. San Fernando, Spain, 1915–1916. (From the Observatory.)

International Nickel Company, Catalogue. New York, New York, no date. (From the Company.)

Jewell Belting Company, Booklet, A Study of Various Types of Belting. Hartford, Connecticut, 1919. (From the Company.)

Jones and Lamson Machine Company, Catalogue on Advanced Practice in Machining Pulleys. Springfield, Vermont, 1919. (From the Company.)

Kingsbury Albert, Catalogue of Kingsbury Thrust Bearings. Pittsburgh, Pennsylvania, 1919. (From Albert Kingsbury.)

Knight, W. B., Machinery Company, Catalogue of Knight Milling and Drilling Machines. St. Louis, Missouri, 1919. (From the Company.)

Lakeside Bridge and Steel Company, Bulletins 201–203, inclusive. 1919. (From the Company.)

Liberty Manufacturing Company, Catalogue "Z." Pittsburgh, Pennsylvania. no date. (From the Company.)

Lietz, A., Company, Catalogue of Drawing Materials and Field Equipment. San Francisco, California, 1919. (From the Company.)

Lehigh University, Register 1918–1919. Bethlehem, Pennsylvania, 1919. (From the University.)

Liverpool Engineering Society, Transactions, Vol. XXXIX. 1918. Liverpool, England, 1918. (From the Society.)

Llewellyn Iron Works, Marine Catalogue. Los Angeles, California, 1919. (From the Works.)

Manganese Track Society, Book 8. Chicago, Illinois, 1919. From the Society.)

Marble-Card Electric Company, Bulletin No. 100. Gladstone, Michigan, 1919. (From the Company.)

Mesta Machine Company, Catalogue for 1919. Pittsburgh, Pennsylvania, 1919. (From the Company.)

Michigan State Board of Agriculture and Experiment Station, Fifty-seventh and Thirty-first Annual Reports from July 1, 1917, to June 30, 1918. East Lansing, Michigan, 1919. (From the Board.)

Mine and Smelter Supply Company, Catalogue No. 42. New York, New York, 1918. (From the Company.)

Miner, J. H., Booklet, The Filers' Success. Lumberton, Mississippi, 1915. (From J. H. Miner.)

Munning, A. P., and Co., Bulletin No. 1000. Philadelphia, Pennsylvania, 1919. (From the Company.)

National Pressed Steel Company, Handbook of National Steel Lumber, Massillon, Ohio, 1919. (From the Company.)

Nevada Railroad and Public Service Commissions, Biennial Reports for 1917–1918. Carson City, Nevada, 1919. (From the Commissioner.)

New Mexico State School of Mines, Catalogue 1918–1919. Socorro, New Mexico, 1919. (From the School.)

New York Conservation Commission, Seventh Annual Report for 1917. Albany, 1919. (From the Commission.)

Nuttall, R. D., Company, Catalogues of Heat Treatment of Steel Gears; Pedigreed Gears and The Nuttall Standard Tractor Transmission. Pittsburgh, Pennsylvania, no date. (From the Company.)

Ohio Mechanics Institute, Catalogue 1919–1920. Cincinnati, Ohio, 1919. (From the Institute.)

Ontario Bureau of Mines, Report No. 17, Parts 2 and 3, 1918. Toronto, Canada, 1919. (From the Bureau.)

Ontario Hydro-Electric Power Commission, Eleventh Annual Report, 1918. Toronto, 1919. (From the Commission.)

Osgood Company, Catalogue of Steam Shovels, Clamshell Outfits and Dredges. Marion, Ohio, 1919. (From the Company.)

Over States Machine Co., Inc., Catalogue, Electric Controlling and Starting Devices. New York City, New York, 1919. (From the Company.)

Patch, F. R., Manufacturing Company, Catalogue K. Rutland, Vermont, no date. (From the Company.)

Pelton Water Wheel Company, Bulletins Nos. 11 and 12. San Francisco, California, 1919. (From the Company.)

Queensland Department of Mines, Annual Report of the Under Secretary for the Year 1918. Brisbane, 1919. (From the Department.)

Roach, Joseph H., and Company, Catalogue, The Roach Stoker. Philadelphia, Pennsylvania, no date. (From the Company.)

Rogers and Hubbard Company, Booklet How to Case-Harden Color and Anneal with Granulated Raw Bone. Middletown, Connecticut, no date. (From the Company.)

Roots, P. H. and F. M., Company, Catalogues Nos. 63 and 68. New York City, New York, no date. (From the Company.)

Ross Heater and Manufacturing Company, Catalogue C. Buffalo, New York, 1919. (From the Company.)

The Shoe and Leather Reporter Company, Annual Directory, 1919. Boston, Massachusetts, 1919. (From the Company.)

Smalley-General Company, Inc., Catalogue, Fundamentals of Thread Milling. Bay City, Michigan, no date. (From the Company.)

Smith, Werner G., Company, Booklet, The Chemistry of a Core. Cleveland, Ohio, 1919. (From the Company.)

South Bend Lathe Works, Lathe Book No. 19, Catalogue No. 57. South Bend, Indiana, 1919. (From the Works.)

Star Brass Works, Bulletin No. 5. Chicago, Illinois, 1919. (From the Works.)

Stuebing Truck Company, Catalogue. Cincinnati, Ohio, no date. (From the Company.)

Tennessee Railroad Commission, Report for the Years 1917–1918. Nashville, Tennessee, 1918. (From the Commission.)

Traylor Engineering and Manufacturing Company, Catalogue of Bulldog Jaw Crushers. Allentown, Pennsylvania, 1919. (From the Company.)

Trimount Rotary Power Company, Booklet of Trimount Rotary Pumps. Boston, Massachusetts, 1919. (From the Company.)

United States Crane Company, Bulletin No. 190. Chicago, Illinois, 1919. (From the Company.)

University of British Columbia, Calendar, Fifth Session, 1919–1920. Vancouver, British Columbia, 1919. (From the University.)

University of Utah, General Catalogue 1919–1920. Salt Lake City, 1919. (From the University.)

Wallace Barnes Company, Booklet of Springs and Screw Machine Products. Bristol, Connecticut, 1916. (From the Company.)

Whiting Foundry Equipment Company, Catalogue No. 145. Harvey, Illinois, 1919. (From the Company.)

Winget, Ltd., Catalogues of the Winget Pressure Machine; The Winget Chain-Spade and the Winget Concrete Block and Slab Making Machine. London, England, 1919. (From the Company.)

Wood, R. D., and Company, Catalogue, The Automatic Gas Producer. Philadelphia, Pennsylvania, no date. (From the Company.)

Vulcan Soot Cleaner Company, Bulletin No. 451. Du Bois, Pennsylvania, 1919. (From the Company.)

BOOK NOTICES.

COBALT: ITS OCCURRENCE, METALLURGY, USES AND ALLOYS. Report of the Ontario (Canada) Bureau of Mines, 1918, vol. xxvii, Part III, Section 1, Prepared by Charles W. Drury. 122 pages and Index, 8vo. Toronto, 1919.

Cobalt has always been numbered among the rather rare metals, and its practical uses have been limited by that fact. According to the report at hand, only five locations are known at which cobalt ores are either being worked or suitable as sources. These are, respectively, in Ontario, Missouri, New Caledonia, Belgian Congo and Schneeberg, Germany. The Ontario deposits, at Cobalt, are the largest. The shipping ore and concentrates contain an average of 7 to 10 per cent. of the metal, 5 per cent. of nickel, 25 per cent. of arsenic, and from 300 to 1000 ounces of silver per ton. As might

be expected from the large amount of arsenic, the cobalt and nickel occur principally as arsenides. Arsenic and silver are not found with the Missouri ores, which are of an entirely different class from the Canadian, containing copper and lead, and much smaller averages of cobalt and nickel. The new Caledonian ore is chiefly oxide, averaging about 3 per cent. The mines of the Belgian Congo yield an ore containing much copper and about 3 per cent. of cobalt. The Schneeberg ores are also poor in nickel and cobalt, but richer in bismuth.

The total world production is about 500 tons per year. The Canadian deposits will be the most prolific source for some years, but the production of the Missouri mines will probably suffice for the needs of the United States. The New Caledonia mines are not active on account of the high cost of transportation.

Over twenty species of minerals containing cobalt as a definite ingredient are described in the pamphlet, and the geology and mineralogy of the deposits in the several localities are extensively treated.

The production of the several metals from the mines at Cobalt, Canada, in 1917, was as follows:

Metal	Tons	Value in Dollars
Nickel	155	125,071
Cobalt	337	1,138,190
Arsenic	2,592	608,483

In addition, 19,401,893 ounces of silver, valued at $16,121,013 were mined. The arsenic is presumably given in the form of arsenous oxide. The total value of the product of the mines during 1917 was $18,028,597.

The uses of cobalt are treated in a special chapter. These were used originally largely in the enamel, porcelain and glass industries, but new uses have been found of late years, among which is an alloy of cobalt and chromium known as "stellite," used in the manufacture of cutting tools. The metal is also added to some high-speed steels. A limited amount of cobalt plating is carried out. The use of cobalt compounds as coloring agents is of great antiquity.

A large amount of space is given to descriptions of the alloys of cobalt with elaborate metallurgic and physical data according to the recent methods in physical chemistry. The pamphlet is well printed and well illustrated.

HENRY LEFFMANN.

THE CONDENSED CHEMICAL DICTIONARY. Compiled and edited by the Editorial Staff of the Chemical Engineering Catalog; F. M. Turner, Jr., Technical Editor; D. D. Berolzheimer, W. P. Cutter and John Helfrich, Assistant Editors. 525 pages, 8vo. New York, The Chemical Catalog, Inc., $5.00 net.

A very large amount of interesting and valuable information is contained in this book. It is almost entirely concerned with the chemical substances used in practical work. Terms in theoretical chemistry are not within its

field. Its scope and method can be best set forth by a brief statement of the treatment of a given item. Opening at random, the reviewer finds on page 54, "Alizarin." The word is starred to show that the substance is now made in the United States. Its systematic name and rational formula are given; the color, melting and boiling points and, in general terms, solubility in alcohol, water and ether. A brief account is then given of the method of preparation. The commercial grades, the material used for containers, the fire hazard and the railroad shipping regulations are stated. In this particular case we learn that there is no fire hazard and the shipment is not specifically controlled. The applications of the "red" "yellow" and "green" labels are indicated whenever necessary. Useful tables of equivalents of Fahrenheit and centigrade scales, and of atomic weights (O = 16 basis), and for converting Baumé degrees into specific gravities, are given; also several pages of definitions of units in the several departments of applied science.

The make-up of the book is excellent—good paper, good type and printing and strong binding. Those engaged in the preparation of the text deserve much praise, and the work will be of great service to practical chemists.

As a second edition of the book will be soon demanded, the reviewer ventures to suggest a further "condensation." Could not melting and boiling points be just as clearly indicated by "b.p." and "m.p."? In many cases the method of manufacture might be omitted, as it is usually too briefly given to be of any practical value. The repetition of titles such as "color," "fire hazard," "railroad shipping regulations," seems unnecessary. As an illustration of the reviewer's idea of condensation, the description of "Nitric Acid" may be considered. This occupies nearly half a page. By condensing such terms as "specific gravity" to "sp. gr." and the other abbreviations just mentioned and omitting the outline of the processes of manufacture, which are much too briefly described to be of any value to the practical worker (elaborate descriptions of such processes are in standard works), a great saving of space could be secured. This would enable a much larger list of substances to be included, and thus increase the usefulness of the volume.

It will be a great convenience to works chemists and commercial analysts to have in compact form and at a reasonable cost such a large amount of information on the properties and composition of commercial chemicals.

HENRY LEFFMANN.

PUBLICATIONS RECEIVED.

How To Do It: A book of "kinks" from the magazine *Concrete*. Compiled by Harvey Whipple, Managing Editor, *Concrete*. 143 pages, illustrations, 12mo. Detroit, Mich., Concrete-Cement Age Publishing Company, 1919.

Technical Books of 1918: A selection. 28 pages, 12mo. Brooklyn, N. Y., Pratt Institute Free Library, 1919.

U. S. Bureau of Mines: Monthly Statement of Coal Mine Fatalities in the United States, July, 1919, compiled by Albert H. Fay. 24 pages, 8vo. Technical paper 239. Coke Oven Accidents in the United States During the Calendar Year 1918, compiled by Albert H. Fay. 26 pages, 8vo. Washington, Government Printing Office, 1919.

U. S. Bureau of Standards: Scientific paper No. 341. Airplane Antenna Constants, by J. M. Cork, assistant physicist. 15 pages, illustrations, 8vo. Washington, Government Printing Office, 1919.

Representation in Industry, by John D. Rockefeller, Jr. Address before the War Emergency and Reconstruction Conference of the Chamber of Commerce of the United States, Atlantic City, N. J., December 5, 1918. 31 pages, 12mo. No place of publication.

The Piezo-electric Effect, a New Principle for Telephone Transmitters and Receivers. A. M. NICHOLSON. (*Session of the Am. Inst. Elec. Engs.*, October 10, 1919.)—When certain crystals are compressed in definite directions a difference of potential is developed between their ends. Conversely, when a difference of potential is applied to the same crystals, they elongate or contract. Similar effects are observed when the crystals are in a condition of torsion. This general effect gives a means of converting mechanical energy into electrical energy and *vice versa*.

The crystals showing the above effect are hemihedral or hemimorphic. Those of Rochelle salt are especially good and large specimens can be grown. Under torsion such crystals give a difference of potential of as much as 600 volts. Desiccated crystals are several times as effective as freshly prepared ones.

The method of transmitting sound is as follows: A crystal under torsion that the motion of the stylus of a phonograph varies the strain already existing and hence develops a difference of potential between the two ends. Two wires are led from the crystal to a 3-stage amplifier to the other end of which is electrically connected a second crystal, likewise under torsion. The varying potential supplied from the amplifier causes the crystal to dilate and to contract. These motions which closely follow the intricate motion of the stylus cause air-waves. Instead of the stylus the human voice may be used directly to vary the force acting on the transmitting crystal.

A demonstration of this very significant method of sound transmission was given in the Bellevue-Stratford Hotel, Philadelphia, October 10, 1919. The voice of a man by "direct action" as well as instrumental music and the voice of Galli Curci from phonograph records were well rendered. The effect was especially good when the receiving crystal was placed on a bare wooden table. It was not even necessary to press the crystal upon the supporting surface. The quality of the sounds produced was good.

This was the first public exhibition of the new means of transmission and was made only about three days after success had been attained in the laboratory.

G. F. S.

CURRENT TOPICS.

The Industrial Value of Mosquito Prevention.—It seems a far cry from the " Promotion of the Mechanic Arts " to the study of the life history of the mosquito, but research has left no reason to doubt the serious injury that this insect can and does do to human beings. In this part of the world malaria and yellow fever are the two diseases which are known to be propagated only by the direct agency of this class of insects. The former is now reduced to minor importance by the thorough measures that have been employed in tropical and sub-tropical countries, but malaria is still a most serious cause of ill-health and consequent loss of efficiency in young adults. No subject is attracting more attention nowadays than industrial hygiene, which in its broadest sense, covers all phases of disease and injury whether directly or indirectly the result of occupation.

Sanitarians have leaned to the view that mosquitoes specialize, so to speak, as disease carriers; that is, one species carries yellow fever and another malaria. *Anopheles punctipennis* has been generally charged with the conveyance of the several forms of malaria, but evidence has been adduced that another species (*A. crucians*) can act as an infection agent. The data are given in a bulletin (Reprint 536 from U. S. Public Health Service Report) by Dr. C. W. Metz. It does not appear that this species is as active as the species above mentioned, as it is apparently an out-of-door biter, not being inclined to enter houses at night. If its infecting power is established the necessity for screening of porches and outhouses will be evident.

In this connection, mention may be made of some experiments carried out under the auspices of the U. S. P. H. Service, by S. F. Hildebrand, Ichthyologist of the U. S. Bureau of Fisheries, to determine the value of fishes as destroyers of mosquito larvæ. The experiments were undertaken in the extra-cantonment zone of Camp Hancock, Augusta, Ga., as a hygienic measure, inasmuch as the district included a large number of shallow collections of water which would serve as mosquito breeding grounds. Many of the collections were drainable, but some were not amenable to this treatment, nor to oiling. The investigation continued from March, 1918, to November, 1918. Several fishes have been suggested as anti-mosquito agents, but Hildebrand limited his work to the common top-minnow (*Gambusia affinis*), which is so well known and so much used as a bait for larger fish. Indeed, this use for bait was one of the interferences with the work, and had to be formally forbidden.

After giving an account of the physio-geographic conditions of the region and the methods of introducing the fish, the results are stated and the following are some of the conclusions: The top-minnow is especially suitable for anti-mosquito work, as it seeks its food at the surface (where the larvæ are), is very prolific, giving birth to well-developed young, and therefore requiring no special hatching or breeding environment; lives and thrives under a wide range of conditions, most of which are especially liable to be mosquito breeding grounds. It can live and multiply in ponds containing predacious fishes, if there is shallow water into which it can escape.

An interesting experiment occurred accidentally. On March 29, a pond was found to be fairly alive with mosquito pupæ and larvæ, but fish not present. Minnows were placed in the water, but all died in less than an hour. The pond was near a sulphuric acid factory. Litmus paper showed strong acid reaction in the water. It appears, therefore, that mosquito larvæ can live in water promptly fatal to fish.

Doctor Metz has also investigated the food of the anopheles larvæ and found that this creature is practically omnivorous, but that it does not thrive in water much fouled with decomposing organic matter. This serves to show that if sloughs and stagnant pools are cleaned, subsequent provision must be made for draining, oiling, fish control or some other method of mosquito eradication.

H. L.

Interesting Technical Exhibits. (*Nature*, July 10, 1919.)—Of especial interest at the British Scientific Products Exhibition in July of this year are these: Apparatus for testing screw-thread gauges by optical projection; a 30-foot range-finder by which a range of 10,000 yards can be signalled to the gun in less than three seconds with an error of twenty-one yards; stages in the development of the airplane, all of metal; model of the largest armor-piercing shell, 18-inch calibre, weighing 1½ tons. In connection with this exhibition Prof. W. H. Bragg lectured on the detection and the transmission of sounds in water. A "hydrophone" was shown which has a diaphragm carying a microphone. On one side the diaphragm is screened against sound waves, while the other side receives them. The vibrations of the microphone are recorded graphically. If a number of stations are equipped with hydrophones and an explosion is produced in the water, the waves will reach the several stations at different times, and from these data and the known speed of sound waves in water the place of the explosion can be worked out, as is done in sound-ranging in air. The method has been applied to distances as great as 230 miles. Thus an additional means of locating ships has been devised.

Another lecture at the exhibition was by H. E. Armstrong on "Coal Conservation," in which he advocated the prohibition of

the use of raw coal in order to substitute the use of smokeless fuel, not only to avoid the smoke but also to save the volatile products. He advised the establishment of large centres of combustion, where the by-products would be utilized and maximum economies perfected. For domestic heating he favored an easily burned fuel in solid form rather than gas.

G. F. S.

Moisture Absorption by Wood Through Varnish. (*Technical Notes, Forest Products Laboratory,* Madison, Wis., 1919.)—In experiments made by the Forest Products Laboratory, it was found that varnishes do not entirely prevent the transmission of moisture into wood, but merely retard it, and that apparently there is no difference in moisture absorption through the coating due to the species of wood used.

The panels used in the experiment were of yellow birch, basswood, red gum, African mahogany, white ash, white pine, Sitka spruce, southern yellow pine, bald cypress, incense cedar, white oak, western yellow pine, Port Orford cedar, and sugar pine.

Three coats of high-grade spar varnish were applied to four panels of each species. Two panels of each species were brush-coated and two were dipped by a special dipping machine designed to secure an even coating. The panels were allowed to dry seventy-two hours between coats and ten days after the final coat before they were given the moisture-resistance test.

The moisture-resistance test consisted in exposing the panels for seventeen days to a humidity of 95–100 per cent., or in an atmosphere practically saturated with moisture.

At the end of this test, it was found that all the brush-coated panels had absorbed between 5 and 6.5 grammes of moisture per square foot of surface, and the dipped panels between 4 and 5 grammes. Such variations in amount of absorption as appeared could easily have been due to inequalities in the application of the varnish. It was quite noticeable that the dipping process produced a more moisture-resistant coating than brushing.

The Dye-Stuff Problem.—In a bulletin (*Tariff Information Series* No. 11) just issued by the United States Government, a large amount of interesting information is given concerning the development and present condition of the dye-stuff supply in this country. It is well known that for many years German dye-stuffs have been in control of the markets of the world, and that dyes of all types have been furnished at low prices and in abundance. Dye manufacturing was carried out in some other countries, even to a limited extent in the United States, but the procedures were largely merely "assembling," that is, bringing together the less complex coal-tar derivatives, commonly called "intermediates," these being in large part imported. A very

interesting and highly important feature of the synthetic dye industry has been the artificial production of certain important natural colors, especially alizarin and indigo. The former was originally obtained only from the madder plant, but its chemical composition and relationships were worked out by German chemists many years ago, and the artificial substance rendered the cultivation of the plant unprofitable. The indigo problem was attacked vigorously, and after a very long and ingenious investigation, was finally solved so that the natural product was driven practically out of the market. Indigo is a typical "vat" dye, that is, the color as it exists in the free state or on the cloth is insoluble in water, but by reduction it becomes colorless and soluble and can then be easily impregnated into the fabric, which by subsequent exposure to air, allows oxidation with the production of the insoluble color. The importance of this particular color is shown by the statistics given in the pamphlet under review. For instance, in 1914 the total importations of indigo (natural and artificial, but presumably almost entirely the latter) were 8,125,211 pounds, of which almost 7,500,000 pounds came from Germany. The total cost was $1,093,266, which is about 12.5 cents a pound. In 1916, 1917 and 1918 no importations from Germany are noted, but it is possible that some of the indigo credited to neutral countries really came from German factories. In 1916, during which the United States was still neutral, about one and one-third million pounds came from England, and over three and one-third million from China. It is probable that much of this from both countries was natural indigo, for the cutting out of the German product enabled the British-India growers to resume cultivation of the plant at considerable profit. The burden imposed by these abnormal conditions is shown by the fact that the indigo imported into the United States in 1916 (nearly 6,600,000 pounds) cost over $8,200,000, about ten times the pound price of the pre-war years.

A special agent has just been sent to Germany to arrange for importation of this and other vat-dyes, which are so much needed by the textile interests of the country. The production of artificial indigo is not a very difficult or complicated process, and in 1917 it was undertaken by an American firm with the result that nearly three hundred thousand pounds were produced that year. In 1918 three firms were engaged in the work and the total product was over three million pounds. It is stated in the report that at present the productive capacity is equal to the total importation for the fiscal year ending June 30, 1914. There is, however, no such record of achievement in the other dyes of the indigo class. Alizarin also, and two alizarin derivatives were made, but in amounts much below the normal demand.

It is gratifying to learn that such progress has been made in the manufacture of certain important synthetic medicinal substances that dependence on Germany is at an end.

Interesting information as to the effect of the synthetic indigo on the production of the natural color is given in a paper by Professor Henry E. Armstrong at a meeting of one of the sections of the Royal Society of Arts (*Jour. Roy. Soc. Arts,* vol. lxvii, p. 446, 1919). When synthetic indigo was first put on the market by the Germans (1897), the area occupied by the indigo culture was 1,688,042 acres. Up to that time the industry had been very prosperous, but the synthetic product at once began to compete, and when the war broke out in 1914 the acreage had fallen to 150,000, about one-tenth the original. The war, of course, by cutting off the opportunities of German competition, gave a chance for advancement of the cultivation of the plant, and by 1917 the acreage had increased to three and one-half times that of the preceding five years. The greatest increase was in Madras, but the encouragement has fallen off materially with the cessation of hostilities, and the production of the natural color is diminishing. In view of what the Americans have accomplished it is likely that the natural product will soon cease to be an important factor in the dye-stuff market. H. L.

An Extension of Applied Chemistry. (*Amer. Chem. Soc. News Service.*)—Department stores and mail order houses are now installing chemical laboratories as part of their organization and are retaining chemists by the year.

One large dry goods firm in a city of the Northwest has a complete laboratory in full view of customers in which tests of all kinds of merchandise are conducted. The exact kind of dye in fabrics, the fibres which they contain, whether cotton or wool or silk, or mixtures of various threads, can thus be determined to the satisfaction of both merchant and consumer.

By having the chemist continually checking up on quality of products, large houses which sell merchandise by parcel post over wide areas can describe exactly everything enumerated in their catalogues. Thus they do not guarantee articles until they have had them thoroughly examined. As customers are so scattered over the country, accurate descriptions of merchandise prevent misunderstandings. The cutting down of unnecessary correspondence is one of the economies which has resulted from taking in a chemical partner.

Much as does big business make use of chemistry at the present time, it was only about a quarter of a century ago that it even sensed the relation of research to success in manufacturing and commerce.

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